# Research on a neutron transport calculation method based on IGA

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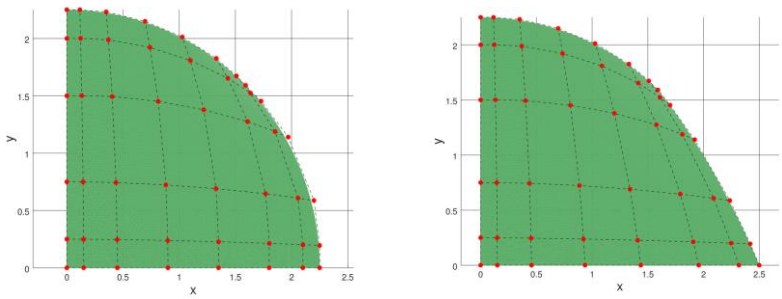
**Abstract** The theory of IsoGeometric Analysis (IGA) allows one to solve the neutron transport equation with a mathematically exact description of the geometrical domain. The IGA method thus does not require any approximations in the description of the geometry. In the current work, the necessary theory for neutron transport theory with IGA is shown and results of a simple test case are reported.
**Keywords:** IsoGeometric Analysis, neutron transport theory, arbitrary geometry, Finite-Element Method

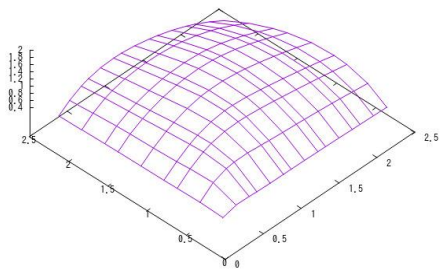
**1. Introduction** The steady-state neutron transport equation is defined on a six-dimensional phase space. In computer simulations, each dimension is discretized. The discretization introduces an error into the calculations. In the current work, Iso-Geometric Analysis (IGA) is applied to the $S_N$ neutron transport equation. IGA allows a mathematically exact treatment of arbitrary geometry, thereby eliminating any errors due to geometrical approximations.

**2. Theory**. In $S_N$ theory, the neutron transport equation is solved on some domain of interested: Cartesian (rectangular) domain, or a cylindrical domain, spherical, etc. A mathematical description of the geometry is needed. In conventional FEM calculations, the domain is approximated (triangles in 2D, hexahedra in 3D), and one must show that the approximation of the geometry does not introduce an unacceptable error. On the other hand, in NURBS theory the description of the geometry is always mathematically exact, i.e. no error is introduced due to geometrical approximation. In Iso-Geometric Analysis, NURBS theory is used to provide a mathematically exact description of the geometry [1]. The NURBS basis functions are subsequently used as basis functions in a FEM calculation based on a classical Galerkin method (Discontinuous Galerkin Finite Element Method, DGFEM). Thus, calculations can be done on arbitrary geometry with good accuracy.

**3. Method development** A calculation method is being developed based on the IGA method using NURBS software developed in earlier work [2]. In Figure 1 an illustration is given of a calculational domain (quarter circle), and a "deformed" domain. Also an example of a solution is given for neutrons entering the domain at an angle of 45º. The power of the IGA approach is that both domains can treated in a mathematically exact way. Thus, effects of thermal expansion can be treated without approximation.




(a) Quarter circle domain with IGA meshing (from [2]) (b) Deformed domain



(c) Solution of the transport equation. Neutrons enter under 45º angle.
Fig 1: calculation domains for IGA and an example solution.

**4. Conclusion** The theoretical framework to solve the neutron transport equation with IGA has been created and a calculation code is under development. Encouraging results have been obtained for several simple test cases. We have already identified a few weak points (most importantly, numerical stability). Another issue is the extension to multi-patch domains, and the fact that boundaries can be re-entrant (i.e. a neutron may leave the domain and re-enter through the same boundary).

**References**

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