Improved Curvature Calculation of the Continuum Surface Tension Model in Moving Particle Semi-Implicit Method

*Jiazhi Li, Sunghyon Jang and Akira Yamaguchi
The University of Tokyo

The curvature calculation for computing the surface tension force is improved in accuracy for particle methods. By means of constructing surface contours around phase interface with phase number fraction, the local curvature of the interface can be obtained with higher accuracy by computing the curvature of the contoured interfaces.

Keywords: curvature, surface tension, particle methods, MPS, multiphase flow

1. Introduction

The moving particle semi-implicit method (MPS) was developed and widely used to simulate incompressible free-surface flows without considering surface tensions. A new method with improved accuracy in curvature calculation based on the continuum surface tension model is proposed.

2. Illustration of curvature calculation

2-1. Methodology

The phase number fraction, denoted as f, is firstly computed for each particle as the ratio of the number of particles in a specific phase to the total number within a searching area with a radius r_s. The radius r_s should be chosen with the magnitude not comparable to the bubble size so that the contoured phase number fraction can be representative for scaled interfaces as shown in Fig. 1. The curvature (κ) calculation can be analytically computed according to the curvature function of implicit curve equations represented by f, which is expressed as follows:

$$\kappa = \frac{2f_{x,i}f_{x,i}f_{x,i} - f_{x,i}^2 f_{yy,i} - f_{y,i}^2 f_{xx,i}}{\left(f_{x,i}^2 + f_{y,i}^2\right)^{3/2}} \quad (1)$$

where right components are the first and second order of derivative functions of f on the Cartesian coordinates, respectively. All derivatives can be obtained by averaging their corresponding gradient vectors between two particles among the searching area. For instance, f_{xx} is given by $\sum_{j\neq i}(f_{x,j} - f_{x,i})(\boldsymbol{r_j} - \boldsymbol{r_i})/|\mathbf{r_j} - \boldsymbol{r_i}|^2$.

2-2. Simulation results

Fig. 2 shows the relative errors of curvature values compared to theoretical ones for a bubble in a circular shape. The absolute relative errors are below 3.8% with sufficient accuracy. A varied r_s is advocated to be used for correctly scaling the interface with different sharpness for higher accuracy.

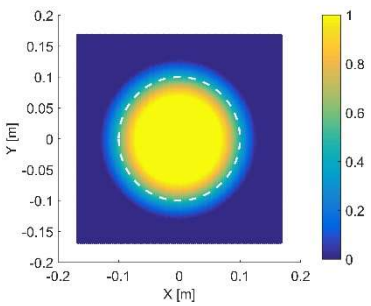

Fig. 1. Phase number fraction.

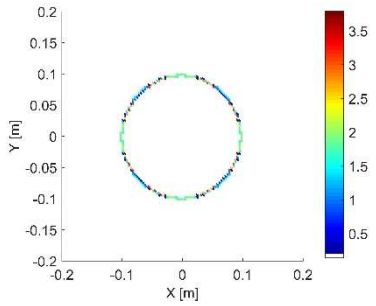

Fig. 2. Relative errors (%) of curvature values.

3. Conclusion

An improved curvature calculation model is proposed for MPS to compute surface tension with higher accuracy.

References

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