Neutron diffusion theory with the IGA method

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Abstract In earlier work the authors presented a solution method for neutron transport theory in two and three dimensions based on the IGA method. In the current work the IGA method is applied to neutron diffusion theory. Diffusion theory requires a different solution method than transport theory. We have successfully implemented a multi-patch diffusion solver based on the IGA method.

Keywords: neutron transport theory, neutron diffusion theory, IGA method, FEM, nuclear reactor simulation

1. Introduction The authors are developing a simulation method for neutron transport theory using the Iso-Geometric Analysis (IGA) method [1]. The IGA method makes it possible to solve partial differential equations (PDE) on arbitrary geometrical domains. The geometrical domain is composed of one or more "patches". In IGA, so-called NURBS basis functions are used to describe the shape of the patches (in two or three dimensions). The same NURBS basis functions are then used to solve a PDE, defined on the patch, using the Galerkin Finite Element Method (FEM). In the current work, the solution method for neutron transport theory based on IGA is extended to neutron diffusion theory. It is intended to use diffusion theory to accelerate the S_N transport calculations.

2. Theory. In our work, the spatial domain is divided into a number of non-overlapping, piece-wise homogeneous patches. On the patches, the PDEs to be solved are:

$$\hat{\Omega}\cdot\nabla\psi\left(\vec{\boldsymbol{r}},E,\hat{\Omega}\right)+\Sigma_{\mathrm{t}}(\vec{\boldsymbol{r}},E)=q\left(\vec{\boldsymbol{r}},E,\hat{\Omega}\right) \qquad \text{Transport theory} \qquad (1)$$

$$-\nabla\cdot D\nabla\phi(\vec{\boldsymbol{r}},E)+\Sigma_{\mathrm{t}}(\vec{\boldsymbol{r}},E)=Q(\vec{\boldsymbol{r}},E) \qquad \text{Diffusion theory} \qquad (2)$$

Compared to transport theory (Eq. (1)), diffusion theory (Eq. (2)) does not contain the variable of neutron propagation $\hat{\Omega}$ and the diffusion equation is a second order PDE whereas the transport equation is a first order PDE. For these types of equations, the boundary conditions and the solution methods are different. In diffusion theory, the boundary condition of zero incoming flux can only be approximated with the concept of extrapolation length. The transport equation can be solved on a patch-by-patch basis using the up-wind method, but for diffusion theory, this method is not available and the solution is calculated for all patches simultaneously, merging the basis functions across the boundaries of the patches.

3. Implementation and Result. It was decided to implement the same method as in the GeoPDEs software package [2], where each basis function has a "local" index on its patch and a "global" index for the global solution. Calculations with external source and multi-group cross sections have shown that our implementation works correctly. However, the implementation of the boundary conditions, especially Dirichlet boundary conditions (i.e. imposed flux on the boundary) requires some more work to obtain a numerically stable solution method.

4. Conclusion. We have established the theoretical framework to implement neutron diffusion theory in 2D and 3D arbitrary geometries using the IGA method. The solution method is accurate. Validation is currently ongoing.

References

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