Distortion Correction and Optical Reconstruction of Point-cloud Object for the Projection-type Color Holographic Display Based on HOE Screen

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ABSTRACT

To reconstruct a desired three-dimensional (3-D) image in the projection-type color holographic display with the holographic optical element (HOE), we shifted coordinates of the point-cloud object theoretically. As a result, we experimentally succeeded in reconstructing the color 3-D image by the optical system including the HOE screen.

1 INTRODUCTION

Holography [1] is a technology that records amplitude and phase information of an object light by interference of light and reconstructs a natural three-dimensional (3-D) image faithfully by diffraction of light. Electro-holography is a technology to record 3-D information of an object on a computer-generated hologram (CGH) and to reconstruct the 3-D image by displaying the CGH on the electronic device such as a spatial light modulator (SLM). However, electro-holography has not been put to practical use. One of the reasons is that the size of the holographic display and the viewing angle for observing the entire 3-D image cannot set independently. Therefore, we cannot observe a large 3-D image at close to the holographic display. When the number of pixels of a CGH is fixed, the relationship between the display size and the viewing angle is determined by Eq. (1).

$$\theta = \sin^{-1}\frac{\lambda}{2p}, \tag{1}$$

where p is the pixel pitch of an SLM displaying a CGH, θ is the maximum diffraction angle of light incident on a CGH, and λ is the wavelength of incident light. The whole size of the display is proportional to the pixel pitch when the total number of pixels is constant. Because we observe the 3-D image through the display, the display area is the observable area. On the other hand, the viewing angle is expressed by 2θ which is inversely proportional to the pixel pitch. Therefore, large observation area and large viewing angle are not realized simultaneously. To solve this problem, a holographic optical element (HOE) [2] written the phase distribution similar to a concave mirror is used as an HOE screen [3][4]. The HOE screen is created by the wavefront printer [5]. By combining a projection lens and the HOE screen with the electro-holography, we can design the screen size and the viewing angle independently. Furthermore, we can determine the observation point arbitrarily in this optical system. In other words, the observation point doesn't have to be on the optical axis. However, the projected image through the HOE screen is distorted when an object is placed far from the hologram plane in CGH calculation. We can observe only the projected image near the hologram plane correctly due to the distortion.

In this study, we calculated a CGH from a point-cloud object whose coordinate was corrected theoretically in advance so as to project the 3-D image at a desired position when it is projected through the HOE screen. In addition, we generated the corrected CGH composed of red, green and blue in order to reconstruct a color 3-D image. As a result, we succeeded in reconstructing the desired color 3-D image in the actual optical system and confirmed the effectiveness of correction.

2 METHOD

2.1 Optical system

Fig. 1 shows a schematic diagram of a simplified optical system including the HOE screen. This optical system is composed of an SLM, a projection lens, and the HOE screen. Table 1 shows meanings of symbols in Fig. 1.

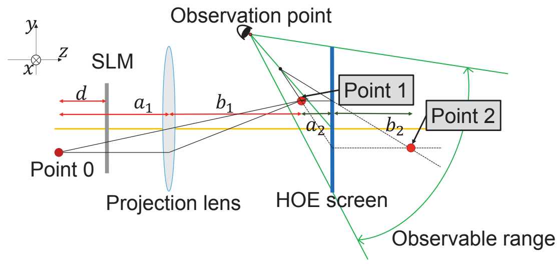


Fig. 1 Optical system including HOE screen in Y-Z plane

Table 1 Symbols in Fig. 1

d	distance between the Point 0 and the CGH plane
a_1	distance between the CGH plane and the projection lens
b_1	distance between the projection lens and the Point 1,
a_2	distance between the Point 1 and the HOE screen
b_2	distance between the HOE screen and the Point 2

We consider the case that a Point 0 is in the left side of the SLM in Fig.1 and the observation point is shifted to the y and z directions in Fig. 1 as follows: Point 0 is one of the object points of the original image; Point 1 is the real image reconstructed by the Point 0 via the projection lens; and Point 2 is the image reconstructed by the Point 1 via the HOE screen. f is the focal length of the projection lens, H_{fz} is the focal length in the z direction of the HOE screen, and H_{fy} is the focal length in the y direction of the HOE screen. We can observe images in the observable range determined by the observation point, the top of the HOE screen and the bottom of the HOE screen geometrically.

2.2 Cause of distortion of projected image

When a Point 0 of the original image in Fig. 1 is close to the hologram surface, the Point 1 is reconstructed as a real image between the HOE screen and the focal point in the z direction of the screen. Here, the Point 2 is projected as a virtual image in the right side of the HOE screen by the HOE screen which works like a concave mirror in Fig 1. In other words, the Point 2 is displayed in the observable range. On the other hand, when a Point 0 is far from the CGH plane, the Point 1 is reconstructed in the left side of the focal point in the z direction of the HOE screen, and the Point 2 is projected as a real image in the left side of the HOE screen in Fig 1. In this case, the Point 2 is displayed outside the observable range. If the depth of a Point 0 becomes even larger, the Point2 is reconstructed as a real image outside the observable range. Therefore, the longer the distance between the original image and the HOE plane is, the larger the distortion of the projected image is.

2.3 Derivation of correction equation

We need to correct the point cloud of the original image in order to observe a desired 3-D image through the HOE screen. Therefore, the mathematical expression of reconstructed position by the optical system is required. The final reconstructed position of the Point 2 in the z direction is derived as follows. First, the lens formula is expressed as

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f}, \tag{2}$$

where a is the distance between the object and the lens, b is the distance between the lens and the image and f is the focal length of the lens. According to Eq. (2), b_1 in Fig. 1 is expressed as Eq. (3) using a_1 and f.

$$b_1 = \frac{a_1 f}{a_1 - f}. \tag{3}$$

Because the HOE screen has the phase distribution similar to a concave mirror, it concentrates incident light to an arbitrarily point. This means that the HOE screen is optically equivalent to a concave mirror. The positional relationship between the object and the image by a concave mirror is shown in Eq. (4).

$$\frac{1}{a}-\frac{1}{b}=\frac{1}{f}, \tag{4}$$

where a and b are the same as in Eq. (2) and f is the focal length of the concave mirror. According to Eq. (4), b_2 in Fig. 1 is expressed as Eq. (5) using a_2.and H_{fz}.

$$b_2 = \frac{a_2 H_{fz}}{H_{fz} - a_2}. \tag{5}$$

By combining Eq. (3) and Eq. (5), the reconstructed position of the Point 2 in the z direction, b_2, can be theoretically derived. In order to correct the position of the point cloud, it is necessary to determine the depth d of the point-cloud object from b_2. The equation used for the correction in the z direction derived from Eq. (3) and Eq. (5) is shown in Eq. (6).

$$d = \frac{f\left(H_{fz}b_2 - d_{LH}\left(H_{fz}+b_2\right)\right)}{(f-d_{LH})\left(H_{fz}+b_2\right)+H_{fz}b_2} - d_{SL}, \tag{6}$$

where d_{SL} is the distance between the SLM and the projection lens, d_{LH} is the distance between the projection lens and the HOE screen.

The final reconstructed position of the Point 2 in the y direction is derived theoretically as follows. The y coordinates of Point 0, Point 1 and Point 2 are y_0, y_1 and y_2, respectively. Equation (7) shows an expression of magnification by a lens.

$$M = \left|\frac{a}{b}\right|, \tag{7}$$

where M is magnification by the lens, a and b are the same as in Eq. (2). According to Eq. (7), the magnification by the projection lens in Fig. 1 is represented by Eq. (8), and the relation between y_1 and y_0 is represented by Eq. (9).

$$M_1 = \frac{a_1}{b_1}, \tag{8}$$

$$y_1 = -M_1 y_0. \tag{9}$$

As previously mentioned, the HOE screen is optically equivalent to a concave mirror. Therefore, the magnification by the HOE screen, M_2, is as shown in Eq. (10) using Eq. (7).

$$M_2 = \left|\frac{a_2}{b_2}\right|. \tag{10}$$

Considering the HOE screen as a concave mirror and geometrically finding the reconstructed position in the y direction by the HOE screen, y_2, is expressed by Eq. (11) using y_1.

$$y_2 = H_{fz} - M_2(H_{fy} - y_1)\,. \tag{11}$$

By combining Eq. (9) and Eq. (11), the final reconstructed position in the y direction can be theoretically derived. It is necessary to determine y_0 of the point-cloud object from y_2 when correcting the position of the point cloud. The function is expressed as

$$y_o = \frac{H_{fy}(1 - M_2) - y_2}{M_1 M_2}\,. \tag{12}$$

3 EXPERIMENTS

To confirm the effectiveness of our correction, we reconstructed a 3-D image from a corrected CGH in the actual optical system including the HOE screen. The optical system is shown at Fig. 2. A laser beam that passed through a mirror and a prism entered the SLM. The SLM displayed a CGH. Then, the light radiated from the SLM passed a 4f optical system consisting of two convex lenses in order to remove the non-diffraction light and the conjugate light. Then, the light entered the HOE screen via a mirror and the projection lens. We observed the 3-D image in the observation point which is previously set.

In this experiment, we reconstructed a color 3-D image using a color CGH including red, green and blue components. 3-D image was photographed by a digital camera installed in the observation point as shown by Fig. 3. Table 2 shows parameters such as distances or pixel pitches used for this experiment. Fig. 4 shows the original object image of the point cloud. The object is a fish which has a red skin, green bones and blue organs. The red skin is composed of 710 points, green bones are composed of 1498 points and blue organs are composed of 1497points. The distance between the center of the 3-D image and the CGH plane was 0.1m.

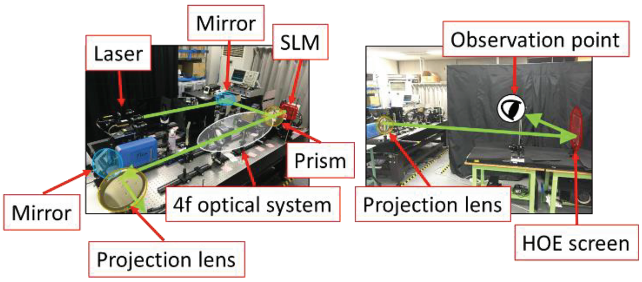


Fig. 2 Actual optical system including HOE screen

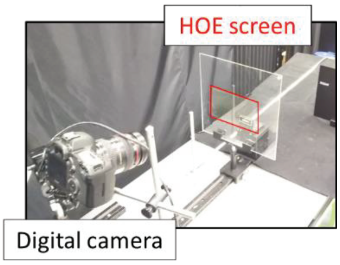


Fig. 3 Digital camera installed in the observation point

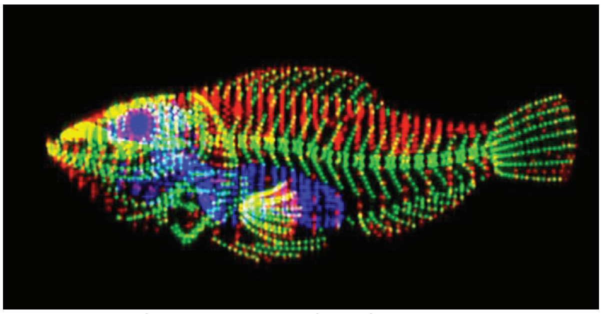
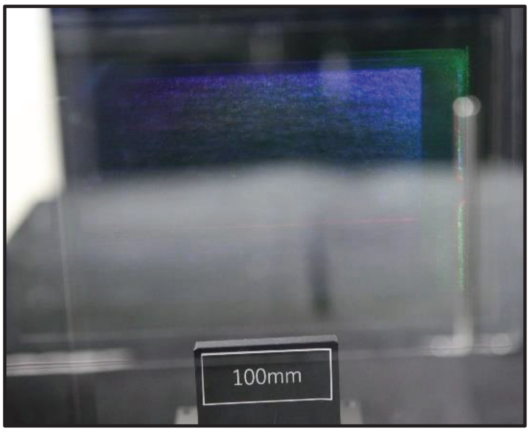


Fig. 4 Original image of the fish in XY plane

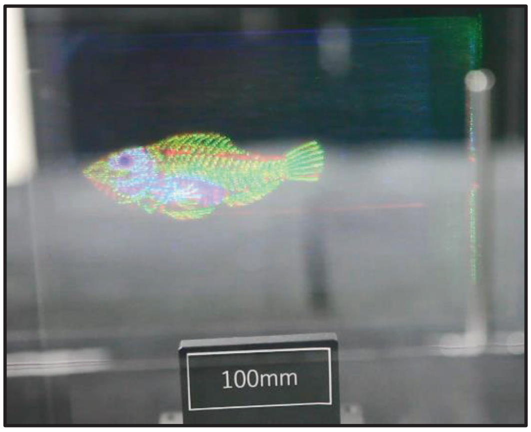
Table 2 Parameters using for the experiment

Wavelength of light [nm]	Red : 660 Green : 532 Blue : 473
Distance between the center of the screen and the observation point [cm]	y : 12 z : 50
Pixel pitch of the SLM [μm]	4.8
Number of pixels of the SLM [px]	7680 × 4320
Distance between the SLM and the projection lens [m]	0.625
Distance between the projection lens and the HOE screen [m]	2.5
Focal length of the projection lens [m]	0.5
Focal length of the HOE screen [cm]	y : 9.6 z : 40

Fig. 5(a) and 5(b) show reconstructed images from CGHs generated by the point cloud without and with correction respectively. A board was placed 0.1m behind the HOE screen. We used this board as a guide of focusing. Fig. 5(a) shows that when the CGH without correction was reconstructed by the optical system including the HOE screen, no image is reconstructed. Fig. 3(b) shows that the same image as Fig 4 was reconstructed with the corrected CGH. Furthermore, it can be confirmed that the correction was effective for RGB colors of light respectively from Fig 5(b). From these results, an appropriate correction was made to the point cloud, and a desired reconstructed 3-D image could be obtained in the optical system including the HOE screen.



(a) Reconstructed image without correction



(b) Reconstructed image with correction

Fig. 5 Reconstructed result

4 CONCLUSIONS

In this research, we treated the object as the point cloud and corrected the point cloud theoretically in advance so that it can be reconstructed as the desired image at the time of projection on an optical system including the HOE screen. Then we created the CGH from the corrected point cloud and confirmed the color 3-D image reconstructed from the CGH using the optical system including the HOE screen. As a result, we confirmed that the color 3-D image through the HOE screen reconstructed from the corrected CGH was undistorted.

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