# Simulation of Target Observation Area Formed by HOE Screen with Function of Concave Mirror

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## ABSTRACT

*When three-dimensional images is reconstructed in projection-type holographic display based on a holographic optical element screen, the observation area is limited to narrow range. In this study, we simulated and evaluated the observation area in order to expand the observation area quantitatively.*

## 1 INTRODUCTION

Holography [1] is a technology that can record and reconstruct three-dimensional (3-D) images by using interference and diffraction of light. In particular electro-holography [2] is a promising 3-D display technology. It can reconstruct 3-D images by displaying the computer-generated holograms (CGHs) on an electronic device such as a spatial-light modulator (SLM) and irradiating them with reference light. However, electro-holography has a problem that it is difficult to realize enlargement of both size and viewing angle of reconstructed 3-D images at the same time because space bandwidth product of commercial SLMs is too low to realize practical holographic display system. To resolve this problem, a screen based on a holographic optical element (HOE) [3], which is called the HOE screen, was developed [4]. In our research, the phase distribution similar to a concave mirror is written on the HOE screen by the wavefront printer [5]. The HOE screen is designed to focus light projected on the screen on the target observation point and form the target observation area around there. Although the observation point is designed freely, the HOE screen has a problem that size of target observation area is limited to narrow range. Then, we aimed to expand the target observation area by shifting the target observation point in time-division multiplexing [6]. In this paper, in order to evaluate the effect of the expansion of target observation area quantitatively, we simulated the optical system with the HOE screen, calculated the size of a target observation area, and examined how much it could be expanded.

## 2 PROJECTION-TYPE HOLOGRAPHIC DISPLAY BASED ON HOE SCREEN

### 2.1 Concept of HOE screen

As discussed in the previous section, electro-holography has the trade off between practical dsplay size and viewing angle. In electro-holography the relationship between the pixel pitch of an SLM and the maximum diffraction angle is written as

$$\theta = \sin^{-1}\frac{\lambda}{2p}, \tag{1}$$

where $\theta$ is the maximum diffraction angle, $\lambda$ is the wavelength of light, and $p$ is the pixel pitch of the SLM. When $\theta$ in Eq. 1 is very small, it is inversely proportional to $p$ approximately. Because the display size depends on space bandwidth product of the SLM and viewing angle depends on the maximum diffraction angle, both large viewing angle and large observation area are not realized at the same time.

Figure 1 shows the basic concept of a projection-type holographic display system based on the HOE screen. The holographic projection consists of electro-holography. A reconstructed image from the SLM is projected on the HOE screen via the projection lens. The HOE screen can design the size of holographic display and the viewing angle independently because the size of holographic display and the viewing angle depend on projection lens in the holographic projection for enlarging the image and the target observation point respectively [3]. Around the target observation point, we can observe a hologram image from the target observation area where all the diffracted light from the HOE screen overlap.

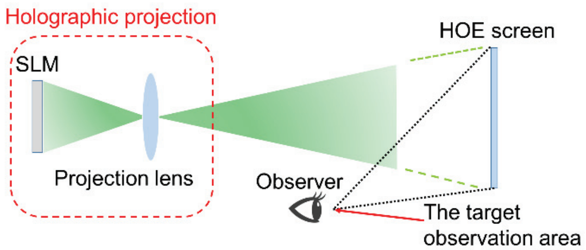


**Fig. 1 Overview of projection-type holographic display based on HOE screen**

### 2.2 Modulation of light reflection direction

If the holographic image is projected on a flat mirror, we cannot observe the entire holographic image because only a small part of the reflection light reaches the observer's pupil. Figure 2 shows modulating the reflection direction of light. The HOE screen made by wavefront printer can have the function which enables us to arbitrarily modulate direction of the diffracted light.

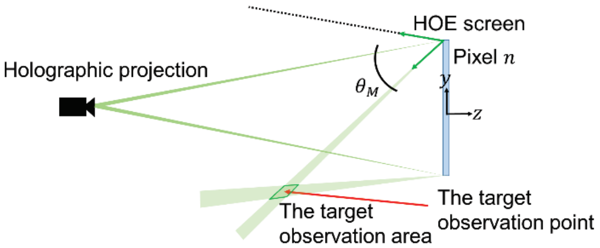


**Fig. 2 Modulating the reflection direction of light at pixel *n***

In this work, we considered only the Y-Z coordinates for simplicity. *n* denotes the Y coordinate of the pixel at the edge of the HOE screen, $\varphi[n]$ denotes the phase distribution of the HOE screen at the pixel, and $\theta_M$ denotes the modulation angles from the specular direction at pixel *n*. Here, the initial value of $\varphi[0]$ is 0. Under the condition of pixel pitch *p* and wavenumber *k*, the phase distribution written on the HOE screen can be calculated by using the recurrence formula as follows:

$$\varphi[n] = \varphi[n-1] + kp\sin(\theta_M). \qquad (2)$$

### 2.3 Target observation area

The light focused on the target observation point forms the target observation area because the light is spread by diffraction. An original hologram image is enlarged and projected through a projection lens at the HOE screen. Figure 3 shows forming the target observation area.

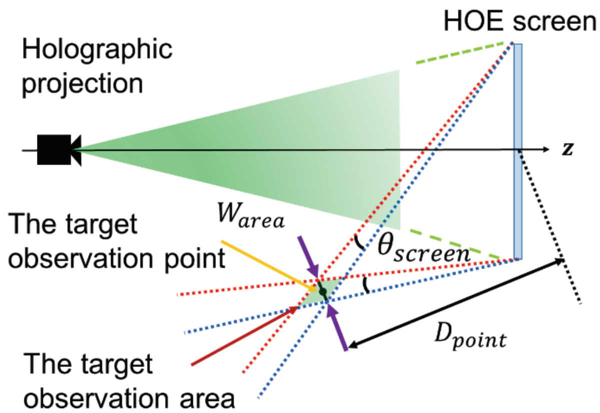


**Fig. 3 Size of the target observation area at the target observation point**

According to Eq. (1), because of the enlargement of a holographic image, the maximum diffraction angle $\theta_M$ at the HOE screen is written as

$$\theta_{screen} = \sin^{-1}\frac{\lambda}{2M_L p}, \qquad (3)$$

where $M_L$ is magnification ratio by a projection lens. We defined the size of the target observation area $W_{area}$ as shown in Fig. 3. $W_{area}$ is written as

$$W_{area} = 2D_{Point}\tan\theta_{screen}, \qquad (4)$$

where $D_{point}$ is a distance between the HOE screen and the target observation point. In this study, it is assumed that the target observation area can be expanded by combining the shifted target observation points by time-division multiplexing.

## 3 NUMERICAL SIMULATION

### 3.1 Simulation model

In this section, we considered an optical system which reconstructs a hologram image with the HOE screen that is twice as large as the SLM. Next, we constructed a simulation model of the optical system based on the parameters of the actual optical system and investigated the target observation area. Figure 4 shows the optical system in numerical simulation. We positioned the target observation point along the $z$ axis for simplicity. In addition, we assumed a transmission-type HOE screen for simulation of the optical system as shown in Fig.4. We used single-side band filter (SSBF) to extract only +1st-order diffraction light from the CGH [7].

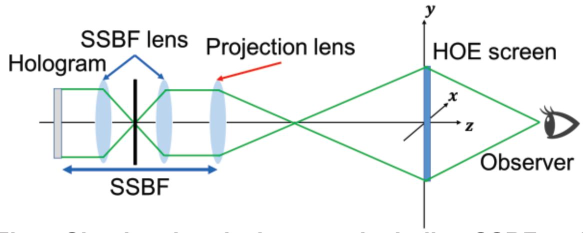


**Fig. 4 Simulated optical system including SSBF and HOE screen**

### 3.2 Results

Table 1 shows the experimental conditions. Figure 5 shows the target observation area which was simulated. The size of the HOE screen was designed depending on the size of the SLM, the pixel pitch of the SLM, and the projection magnification. In this study, we simulated the entire target observation area. Figure 6 shows the results of the target observation area at each target observation point and Table 2 shows the values of each target observation area obtained by Fig. 6. Also, output images in Fig. 6 were normalized to 256 greyscale gradations.

**Table 1 Experimental conditions**

| | |
|---|---|
| Size of HOE screen | 3840×2160[pixel] |
| Size of SLM | 1920×1080 [pixel] |
| Pixel pitch of SLM | 8.5 [μm] |
| Projection magnification | 2.0 |
| Maximum diffraction angle of SLM | ±1.79 degree |
| Wavelength | 532 [nm] |
| Focal length of projection lens | 400 [mm] |
| Focal length of SSBF lens | 300 [mm] |

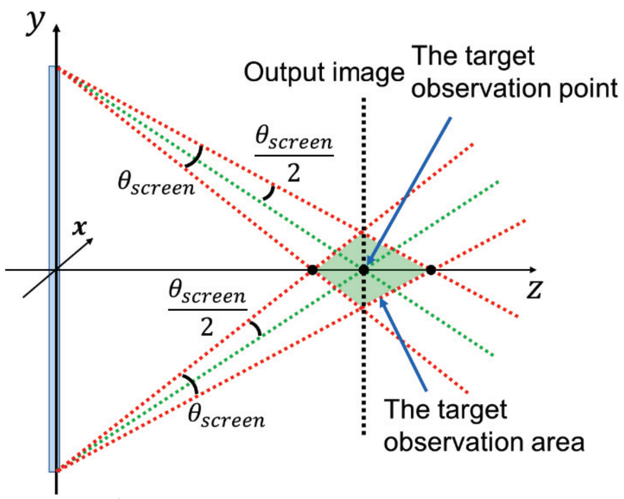


**Fig. 5 Output image simulated at the target observation area**

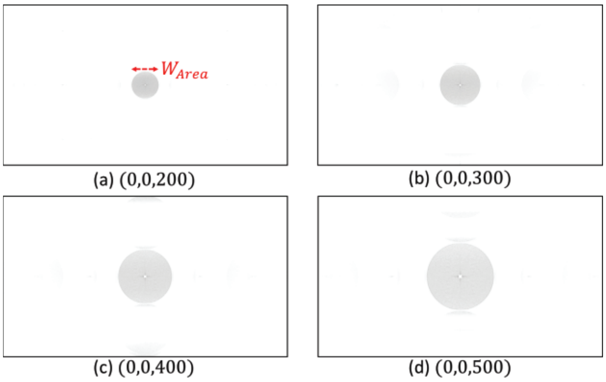


**Fig. 6 Simulation results of the target observation area at the coordinates of each target observation point of (a) (0, 0, 200), (b) (0, 0, 300), (c) (0, 0, 400), and (d) (0, 0, 500)**

**Table 2 The size of each target observation area at the coordinates of each target observation point**

| The coordinates of the target observation point $(x, y, z)$ [mm] | $W_{Area}$ [mm] |
|---|---|
| (0,0,200) | 6.21 |
| (0,0,300) | 9.18 |
| (0,0,400) | 12.28 |
| (0,0,500) | 15.34 |

The size of the target observation area at the target observation point depends on the maximum diffraction angle of the HOE screen and distance from the HOE screen. According to Eq. (3) and (4), the size can be calculated from the coordinate of the target observation point and the maximum diffraction angle of the HOE screen. The shape of a target observation area is circular on the vertical surface. The maximum diffraction angle at the HOE screen is

$$\theta_{screen} = \sin^{-1}\left(\frac{532 \times 10^{-9}}{2 \times 2 \times 8.5 \times 10^{-6}}\right) = 0.90. \tag{5}$$

For example, the diameter of the target observation area in Fig. 6 (a) is

$$W_{area} = 2 \times 0.2 \times \tan(0.9) = 6.28[\mathrm{mm}]. \tag{6}$$

Thus, the target observation area in Fig. 6 (a) is about $31.0\ [\mathrm{mm}^2]$.

## 4 CONCLUSION

In this paper, we simulated the target observation area at the target observation point. We generated the images of the target observation area in Fig.6 and succeeded in investigating the size and shape of the target observation area.

In future works, we will consider combining the target observation area in time-division multiplexing by using inline and off-axis reconstruction and expanding the target observation area. To expand the target observation area to the fullest extent possible, we make the target observation area properly adjacent to each other by simulation.

## ACKNOWLEDGMENT


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