Optimization Technique for Phase-Only Computer-Generated Holograms Based on Gradient Descent Method

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Keywords: Holography, Fast calculation, Algorithm

ABSTRACT

A new optimization technique for phase-only computer-generated holograms is proposed, which iteratively updates the phase distributions based on the gradient of the root mean square error of the reconstructed images. The number of iterations in the optimization process required for the proposed technique is much less than that required for the Gerchberg-Saxton algorithm.

1 INTRODUCTION

Holography [1,2] is a technology that records and reconstructs the wavefront emitted from three-dimensional (3D) objects. When holograms are calculated using a computer, the computer-generated holograms (CGHs) can produce 3D images from 3D data stored in a computer [3-6].

The phase-only holograms can produce bright 3D images because the amplitude of light is not modulated. However, the phase distributions cannot be obtained by simply calculating the light diffraction from 3D objects. The conventional algorithm is the Gerchberg-Saxton (GS) [7] algorithm, which is known as an effective phase optimization algorithm, and recently, Mixed Region Amplitude Freedom (MRAF) algorithm [8] which is improved algorithm of the GS algorithm and Offset MRAF (OMRAF) algorithm [9] which is improved algorithm of the MRAF algorithm are proposed. However, the reconstructed images generated by the GS algorithm still contain errors. Moreover, the optimization speed of the GS algorithm reduces with increasing the number of iterations.

In this paper, a new algorithm based on the gradient descent method is proposed. The mean square error (MSE) is used to optimize the phase distributions.

2 THEORY

When the intensity distribution of reconstructed image is denoted by I_{mn} and the target intensity distribution is denoted by I_{mn}, the MSE of the reconstructed image is given by:

$$E = \frac{1}{MN}\sum_{m}^{M}\sum_{n}^{N}\left(\hat{I}_{mn} - I_{mn}\right)^2 \tag{1}$$

where the resolution of holograms is represented by MXN. When the phase of the phase-only hologram is represented by θ_{pq}, the gradient of the MSE regarding to the phase distribution is given by:

$$\frac{\partial E}{\partial \theta_{pq}} = -4\, Im\left\{e^{-j\theta_{pq}}\mathcal{F}^{-1}\left[\left(\hat{I}_{mn} - a_{mn}^2\right)a_{mn}e^{j\phi_{mn}}\right]\right\} \tag{2}$$

where a_{mn} and ϕ_{mn} are the amplitude and phase distributions of the reconstructed image. Thus, we update the phase distribution using the equation shown below:

$$\theta_{pq}^{t+1} = \theta_{pq}^{t} - \gamma_t \frac{\partial E}{\partial \theta_{pq}} \tag{3}$$

where t is the iteration number and γ_t is the step length coefficient.

There might be many choices for the determination of the step length coefficient. Here, we determine the coefficient using the estimation of the next MSE. The next iteration's MSE can be presented as below.

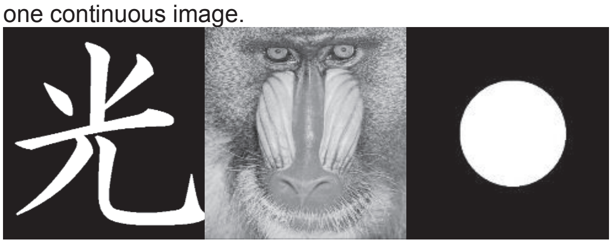
$$E^{'} = \sum_{m}^{M}\sum_{n}^{N}\frac{\left[\hat{I}_{mn} - \left(I_{mn} + \Delta I_{mn}\right)\right]^2}{MN} \tag{4}$$

where ΔI_{mn} is the estimated direction determined by $\Delta\theta_{pq}$. To minimize E', the coefficient $\gamma$is obtained from $\frac{dE^{'}}{d\gamma} = 0$.

$$\gamma = \frac{\sum_m^M \sum_n^N \left(\hat{I}_{mn} - a_{mn}^2\right) Im\left\{a_{mn}e^{j\phi_{mn}}\left[\mathcal{F} - \frac{\partial E}{\partial \theta_{pq}}e^{j\theta_{pq}}\right]^*\right\}}{2\sum_m^M \sum_n^N \left\{Im\left\{a_{mn}e^{j\phi_{mn}}\left[\mathcal{F}\left(-\frac{\partial E}{\partial \theta_{pq}}e^{j\theta_{pq}}\right)\right]^*\right\}\right\}^2} \tag{5}$$

3 CALCULATION OF PHASE DISTRIBUTIONS

The phase-only holograms were calculated using the proposed algorithm. Figure 1 shows the target reconstructed images including two binary images and one continuous image.

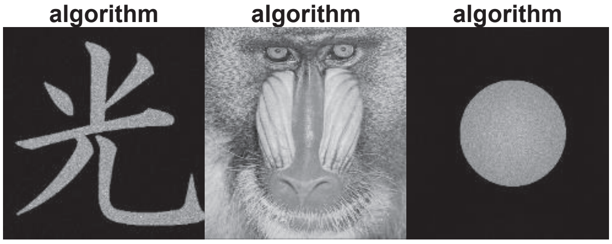


(a) "hikari" (256×256) **(b) "mandrill" (512×512)** **(c) "circle" (512×512)**

Fig. 1 Target reconstructed images

Figures 2(a1), 2(b1) and 2(c1) show the reconstructed images generated by the phase distributions obtained by the proposed technique. The number of iterations was 100. For comparisons, the reconstructed images obtained by the GS algorithm are also shown in Figs. 2(a2), 2(b2) and 2(c2).

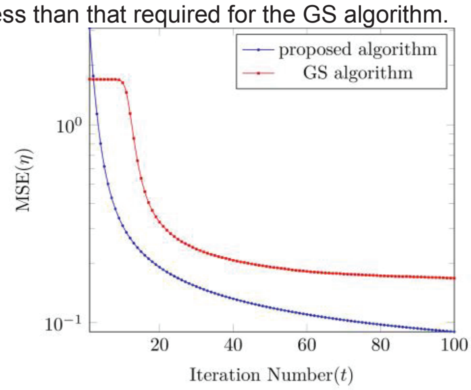
(a1) “hikari” by proposed algorithm | (b1) “mandrill” by proposed algorithm | (c1) “circle” by proposed algorithm



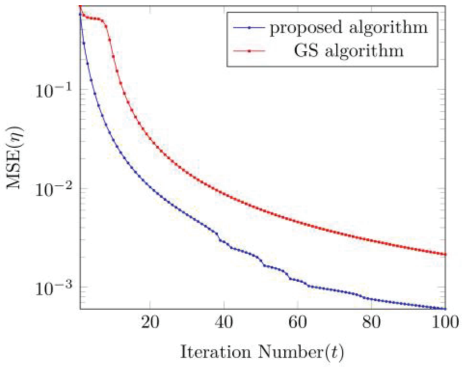
(a2) “hikari” by GS algorithm | (b2) “mandrill” by GS algorithm | (c2) “circle” by GS algorithm

Fig. 2 Reconstructed images generated by the phase distributions

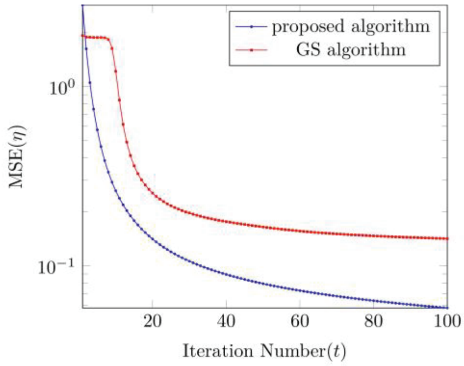
Figure 3 shows the changes of the MSEs during the optimization processes. The proposed technique provided smaller MSEs than the GS algorithm. The number of iterations required for the proposed technique was less than that required for the GS algorithm.



(a)“hikari”



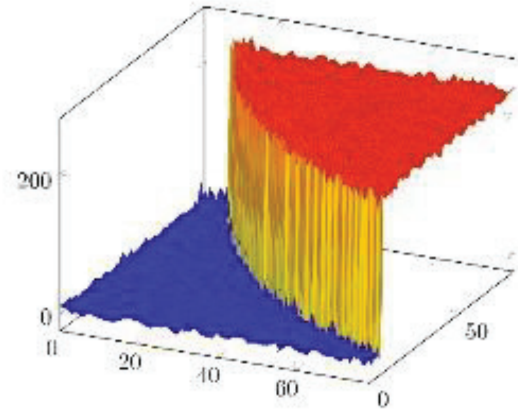
(b) “mandrill”



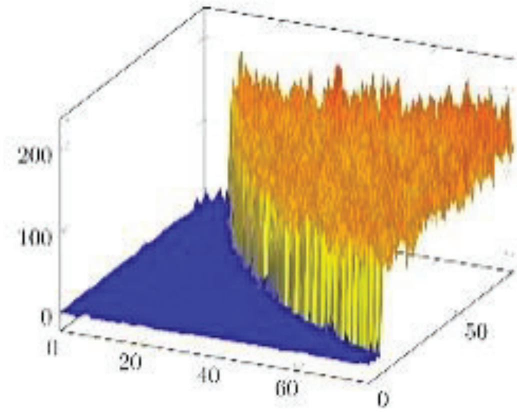
(c)“circle”

Fig. 3 Changes of the MSEs during the optimization processes

Figure 4 shows the intensity distributions of the magnified reconstructed images shown in Fig. 2. The proposed technique controlled the intensity distributions more precisely than the GS algorithm.



(a) Intensity distributions of the magnified reconstructed images shown in Fig. 2(c1)



(b) Intensity distributions of the magnified reconstructed images shown in Fig. 2(c2)

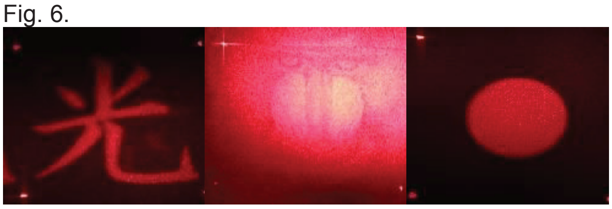
Fig. 4 Intensity distributions of the magnified reconstructed images shown in Fig. 2

4 EXPERIMENTS

The reconstructed images of the phase-only CGHs calculated in the previous section were experimentally obtained. We constructed the optical system shown in Fig. 5.

Fig. 5 Practical Optical System

The phase-only SLM (PLUTO-2, HOLOEYE Photonics AG) was used to display the hologram patterns. The He-Ne laser with a wavelength of 633 nm was used as a light source. We captured the reconstructed image using a video camera. The reconstructed images are shown in Fig. 6.



(a) “hikari” (b) “mandrill” (c) “circle”

Fig. 6 Optical reconstructed images of holograms generated by proposed algorithm

5 CONCLUSION

We proposed a new algorithm to optimize the phase-only CGHs. The proposed algorithm optimized the phase distribution faster than the GS algorithm and provided more precious reconstructed images than the GS algorithm.

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