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***(Late-News Paper)***

# Analytical Solution for three Conjugates Vari-Focal Based Augmented Reality System

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## ABSTRACT

*In order to design the compact Augmented Reality (AR) system within various design configurations, an analytical solution is essential to explore the limitations of the design space parameters on the system performance metric. This article reveals a compact analytical form for the three conjugate AR configurations with vari-focal lens (VFL) time multiplexing capabilities. This VFL-AR equation can describe the first-order system metric, which can find the feasibility and the efficiency for further complicated optimizations criterion during the ray tracing.*

## 1. Introduction

AR/VR is getting much more attention toward commercialized targeting market than only the Proof-of-Concept (POC) demonstrations. It is no doubt that a robust system design can stay its high qualities and performance, once the robust parameters, which near the sweet point for sure, is conducted and proven with *grounded physical* sense. Among various type of AR/VR system, vari-focal based system, offer a possible solution to overcome both the VAC (Vergence accommodation conflict) issue and the Cost/Performance requirements for the market. **Figure 1(a,b)** give the principle for the vari-focal system **[1, 2]**.

A good design required the complete compatible on the physical quantities between the key components and the system architectures. Moreover, a robust design required a stability (tolerating) analysis for these conducted qualities. Therefore, it is the demands to start the design of the VFL-AR system that consider in details the design parameters through the mathematical (physical) description directly, rather to set up a guessing parameters in the design software under time-consuming iterations. We believed that the computational time for layout optimization by the brute force method *can* be reduced, if self-consist initial parameters are given in a proper way.

In this report, we will introduce the derivation of the basic algebraic equation for designing the VFL-AR system, and then a compact form will be complete for the designing purpose. The design space could be easily reveals through the present strategies, and design examples are presented to discuss the effectiveness of the present methodologies.

## 2. Theory

### 2. 1 conjugate equations

In this section, we explain the derivations of the named VFL-AR equation. The basic concept is that display panel (generally the micro-display), vari-focal lens, eye box and virtual images must satisfy defined conjugates relationship **[3]**.

**Figure 2** shows the first conjugation. This conjugation indicates the mapping of the object on the micro-display into an intermediate image. Thus, the followings first order mathematics stands in Gaussian form:

$$\begin{aligned}
&\frac{1}{O_1}+\frac{1}{I_1}=\frac{1}{f_1}\\
&\frac{1}{f_1}=\frac{1}{f_{1G}}+\frac{1}{f_{VFL}}-\frac{d}{f_{1G}\cdot f_{VFL}}\\
&m_1=-\frac{I_1}{O_1}=-\frac{h_{imd}}{h_{OLED}}\\
&I_1=(1-m_1)f_1
\end{aligned} \tag{1}$$

Now, **figure 3** gives the second conjugate in which construct the connections between the aperture of the vari-focal lens with the eye box:

$$\begin{aligned}
&\frac{1}{L}+\frac{1}{L'}=\frac{1}{f_{FFL}}\\
&m_2=-\frac{18}{L}=-\frac{h_{eyebox}}{h_{VFL}}\\
&L'=(1-m_2)f_{FFL}
\end{aligned} \tag{2}$$

Finally, **figure 4** give the last conjugation, which complete the projecting of the intermediate image into the virtual position (i.e. AR objects layers) through various kinds of freeform lens or proper combiners.

$$\frac{1}{P}+\frac{1}{P'}=\frac{1}{f_{FFL}}$$
$$m_3=-\frac{P'}{P}=-\frac{h_{lay}}{h_{imd}}$$
$$P'=(1-m_3)f_{FFL} \tag{3}$$

Equation sets (1)-(3) describe the nominal type of vari-focal based AR system architecture. It will be a rational action to combine all these equations to look for a more compact mathematical form. To complete this, constrains is required - as building the structure compatibility and the energy continuities between the three independent conjugates systems for the VFL-AR configurations.

## 2.2 constrains

A feasible design must meet the basic mechanical constrains, which is depend on the topological structure of the system. Here give one simple constrains that show the relationship between the three conjugates:

$$I_1-L+P=\Delta \tag{4}$$

$\Delta$ indicate a given tolerance for system layout (mechanical tolerances, lens positions, packaging ect.). After conducting the first order mathematical descriptions for all the conjugates with this constrain equation, algebraic equations set (1-4) can lead to an initial design among various design parameters. Note that the number of design variables and the number of equations is ***not*** identical, tells that the present equation set is undetermined and the multiples solutions are possible.

## 2.3 AR equation

Consider the following designing methodologies for the initial VFL-AR system.

| | |
|---|---|
| (1) | Design variable：$f_{1G}$, $O_1$,d, |
| (2) | Control parameters：$f_{VFL}$, P’ |
| (3) | Given constant：$h_{VFL}$, $h_{OLED}$, L’ |
| (4) | Target Specification：$h_{lay}$(P),$h_{eyebox}$, $f_{FFL}$ |

After some manipulation **[4]**, one can get the following compact equation (5) that can describe the contextual from figure 2 to 4, as well as the implied light field mechanism shown in **figure 1**.

$$P=\frac{-(I_1-\Delta)+\sqrt{(I_1-\Delta)^2+\dfrac{4(I_1-\Delta)}{\left(\dfrac{1}{L'}-\dfrac{1}{P'}\right)}}}{2}$$ ,where
$$I_1=\frac{1}{\left(\dfrac{1}{f_{1G}}-\dfrac{1}{O_1}\right)+\dfrac{1}{f_{VFL}}\left(1-\dfrac{d}{f_{1G}}\right)} \tag{5}$$

Equation (5) is the named VFL-AR equation.

## 3. Results and Discussion

Implementation of VFL-AR equation can be as follows:

(1) Given the control parameters for the focal length $f_{VFL}$ of varifocal lens and the desired virtual layer position P’

(2) Assign the design constant of aperture of varifocal lens $h_{VFL}$, size of micro-display $h_{OLED}$ and eye relief L’

(3) One can adjust the corresponding design variable: focal length of lens group $f_{1G}$, distance between micro-display and group lens/vari-focal lens system $O_1$, distance between group lens and vari-focal lens d

(4) Check whether or not the conducted values meet the targeting design specification: Size of virtual image $h_{lay}$(P) at position P, size of $h_{eyebox}$ and the affordable freeform focal length $f_{FFL}$.

(5) Repeat (1)-(5) until the satisfactions for both optical structure and mechanical layout

Base on the above approach, one can obtain the results in a blink of an eye. **Table.1** provide one set of initial parameters combinations (not optimized) by the VFL-AR Equation. Note that this is not the optimized results, and it required ray tracing to complete the design procedure. However, due to the under-determined properties for the equations set as mentioned before, the present method can give a very quick estimate at the initial design stage. In advance, designer can define specific constrains to improve the evaluation, and confirmed C/P index for their targeting market.

## 4. Conclusions

We demonstrate the possibilities on the designing of the AR system through systematic optimizations of derived VFL-AR master equation. To give general description for different mechanical layout design, dimensionless procedure is working on for the generalized description for the master equation. Details will be revealed on site and in the upcoming publication.

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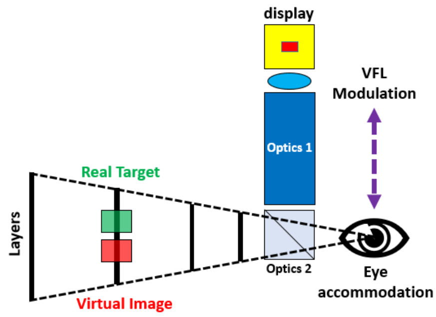


**Fig.1 (a) Vari Focal Lens system [1]**

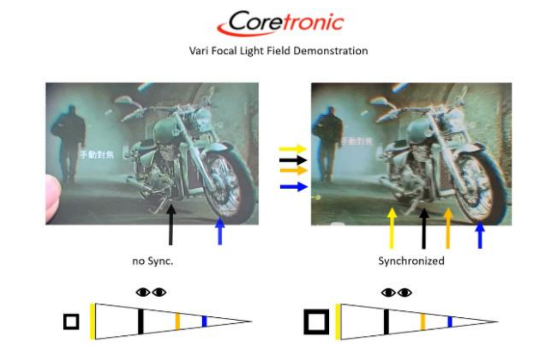


**Fig.1 (b) Vari Focal Lens System Demonstration**

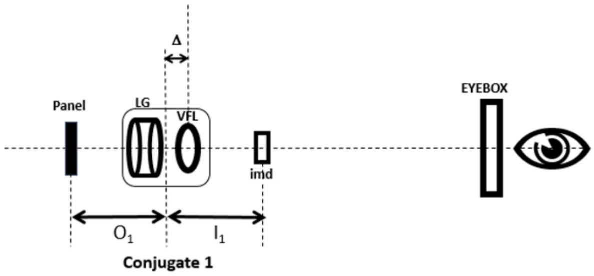


**Fig. 2 conjugate between panel and intermediate image**

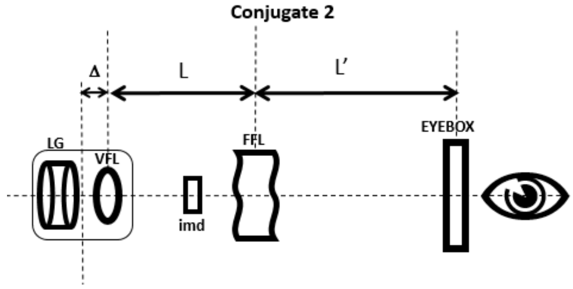


**Fig.3 conjugate between aperture of Vari-focal lens and eye-box**

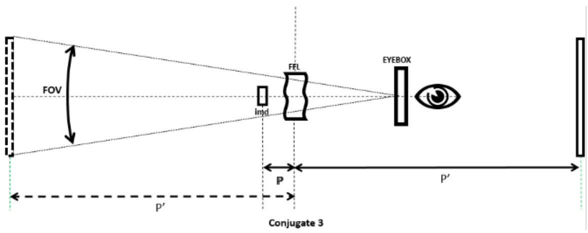


**Fig.4 conjugate between intermediate image and virtual image for AR/VR Layer(s)**

**Table.1 VFL-AR Equation solution example**

| Variable | Values | units | notes |
|---|---|---|---|
| f1G | 2.000 | mm | |
| O1 | 5.000 | mm | |
| d | 1.000 | mm | |
| fVFL | 100 | mm | |
| P' | 500 | mm | |
| hVFL | 3.300 | mm | Optotune |
| hOLED | 4.953 | mm | 0.39" BOE |
| L' | 18.000 | mm | |
| Delta | 1 | mm | |
| L | 7.761 | mm | |
| I1 | 3.279 | mm | |
| P | 5.482 | mm | |
| Hlay | 296.214 | mm | |
| Heyebox | 7.654 | mm | |
| FOV | 59.525 | Degree | |
| FFL | 5.423 | mm | 1/L+1/L' |
| | 5.423 | mm | 1/P+1/P' |