# Factors Affecting the Modulation Transfer Function (MTF) in Polarized Aerial Imaging by Retro-Reflection (p-AIRR)

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Email: hirotsugu@yamamotolab.science Utsunomiya University, Yoto 7-1-2, Utsunomiya City, Tochigi 321-0904, Japan Keywords: aerial image, modulation transfer function, AIRR, p-ARR, retro-reflective.

### ABSTRACT

We will elucidate the factors that influence the MTF of aerial imaging by retro-reflection using polarization modulation (p-AIRR). The MTF decreases as the angle of incidence increases and the floating distance increases. Polarization rotation angle change has no significant influence on MTF.

# **1 INTRODUCTION**

Various aerial display has been studied because of their merits, such as hand-reaching information in the air, a high sense of presence and hygienic because of no physical contact. Aerial imaging by retro-reflection (AIRR) is a method of forming aerial images that can be seen from a wide viewing range [1]. Polarized AIRR (p-AIRR) has been proposed to improve the luminance of aerial images [2]. Polarization modulation forms a bright aerial image that can be observed even on the street.

Along with luminance, resolution is an important factor for quantifying the performance of the optical system for aerial imaging. Regarding the resolution of aerial displays, a quantification method using a transfer function (CTF) that shows the contrast value by changing the width of the black and white striped pattern has been reported [3]. For CTF, the contrast performance of the display itself, which is the light source for aerial imaging, affects the CTF of the total system, so it is not suitable for quantifying the imaging performance of the optical system part of aerial imaging.

In recent years, many optical elements for aerial displays have been proposed as not only retroreflective elements used in AIRR, but also micro-optical elements such as two-sided corner reflector arrays and slit mirror arrays [4]. In these micro-optical elements, since the wavefront is inverted in the unit optical element, the image is formed by focusing the light rays instead of adding the wavefronts, which is different from the image formation by the conventional lens. Such retro-reflective or micro-optical elements cannot provide an image formation in a strict sense according to Abbe's imaging theory, and the international standardization for quantification of imaging performance has not yet been determined. These make it difficult to establish the resolution estimation method.

We have proposed a method for measuring the modulation transfer function (MTF) based on the obligue knife edge method in order to quantify the resolution of aerial imaging by AIRR [5]. In the proposed method, an aerial image is taken by blocking the omnidirectional illumination with a knife edge using an integrating sphere, noise is removed by wavelet transform, edge direction is detected by Radon transform, and then the direction orthogonal to the edge. The line spread function is obtained from the derivative of the edge spread function and the MTF is obtained from the Fourier transform the line spread function. Based on the proposed method, we clarified the effectiveness of the proposed method and the MTF measurement of conventional AIRR using the corner cube type and ball lens type retroreflective elements [6]. This leads that the floating distance, anisotropy, and incident angle affect the prism-type retroreflective element as factors that affect the MTF of the aerial image in conventional AIRR. However, in p-AIRR, what factors affect the MTF have not been reported yet.

In this study, we clarify the factors that affect the MTF of aerial imaging in retroreflection (p-AIRR) using polarization modulation by changing the angle of incidence, angle of rotation, and floating distance to measure MTF.

### 2 PRINCIPE OF AERIAL IMAGING BY RETRO-REFLECTION (AIRR)

The principle of p-AIRR is shown in Fig.1. The p-AIRR consists of a light source, a reflective polarizing plate,  $\lambda / 4$  wavelength film, and a retroreflective element. The reflective polarizing plate reflects S-polarized light and transmits P-polarized light. The  $\lambda / 4$  wavelength film gives a phase delay of  $\pi / 2$  with respect to the electric field component in the slow phase axial direction.



In Fig. 1, the light source emits linearly polarized light, and the polarization direction of the light source and the reflective polarizing plate are in a cross-Nicole arrangement. The linearly polarized light emitted from the light source is reflected by the reflective polarizing plate. The reflected light is given a phase delay of  $\pi$  / 2 by passing through the  $\lambda$  / 4 wave plate, and is changed to circularly polarized light. The circularly polarized light is retroreflected into the incident direction. The retroreflected light of circularly polarized light is changed to opposite rotation state of circularly polarized light because of changing light direction. This leads that retro-reflected light has orthogonal polarization to the original linearly polarized light by the  $\lambda / 4$  wave plate. The light transmitted through the  $\lambda$  / 4 wave plate for the second time is transmitted through the reflective polarizing plate because it is parallel to the transmission axis. The transmitted light is focused on the reflective polarizing plate at a position symmetrical to the light source to form an aerial image.

In the conventional AIRR that does not use polarized light, light loss occurs during reflection and transmission by the beam splitter, whereas in p-AIRR, the reflectance is high before retroreflection and the transmittance after retroreflection. As the height increases, the efficiency of light utilization increases. In other words, the aerial image formed by p-AIRR can form a brighter aerial image.

# 3 PLINCIPLE OF SLANTED KNIFE EDGE METHOD FOR MTF MESUREMENT

MTF expresses how finely the contrast of the subject can be reproduced as a spatial frequency characteristic. The slanted knife edge method estimates the MTF curve for spatial frequency in one-direction by calculating a region of interest (ROI) in a recorded edge image.

The process of slanted knife edge method is shown in Fig. 2. The sampling rate can be improved by slightly slanting the edge image horizontally or vertically. As shown in Fig. 2(a), ROI is extracted from the slanted knife edge image and converted to an 8-bit grayscale image. Fig. 2(b) shows the projection method of edge spread function (ESF) curve. ESF curve is obtained by projecting data of slanted edge images specified by ROI onto the projection axis and superimposing it. By binning the obtained projection data,

obtained ESF curve can increase the sampling interval. The obtained ESF curve is eliminated the problem of aliasing. Fig. 2(c) shows a conceptual diagram of the obtained ESF curve, which contains noise like line thickness. Obtained ESF curve is denoised using wavelet. Fig. 2(d) shows the conceptual diagram of ESF curve after denoising. The line spread function (LSF) curve is derived by differentiating the denoised ESF curve. Fig. 2(e) shows a conceptual diagram of LSF curve. The MTF curve shown in Fig. 2(f) is derived by performing a Fourier transform on the LSF curve, normalizing it, and taking the absolute value.



(a) 8-bit grayscale image selected by ROI.



(b) Projection method of slanted edge image.



# 4 EXPERIMENTAL SYSTEM FOR MTF MEASUREMENT

### 4.1 Optical system setup

Figure 3 shows the optical system for measuring MTF in p-AIRR. A reflective polarizing plate (custom order) and a prism type retroreflective element (Nippon Carbide

Industries: RF-Ax) were used to form the aerial image, and a  $\lambda$  / 4 retardation film was attached to the retroreflective element for p-AIRR. A digital camera (Nikon, D5500) installed at an angle of 5 degrees with respect to the axis of the knife edge was used to shoot aerial images. The recording conditions of the camera were as follows: ISO sensitivity 400 of the image sensor, the focal length of the lens of 35 mm, the F value of 4.5, and the exposure time of 1/60 second.

Figure 4 shows the photographs of aerial images formed by (a) AIRR and (b) p-AIRR under the same conditions. There are aerial images of a part of the opening of the integrating sphere blocked by a knife edge. Comparing these aerial images, the aerial image formed by p-AIRR is much brighter than that by conventional AIRR.



Fig. 3 Experimental setups for MTF measurements.



(a) Aerial image formed by conventional AIRR.



(b) Aerial image formed by p-AIRR. Fig. 4 Photographs of aerial images of a part of the opening of the integrating sphere blocked by a knife edge

### 4.2 Parameter setup

In this experiment, three parameters of the incident angle, polarization rotation angle, and floating distance are changed.

# 4.2.1 Angle of incidence on the retro-reflective element

For investigating the influence that the rotation of polarized light in retardation film does not reach 90 degrees in an oblique incidence, the MTF was measured by changing the angle of incidence of the light beam on the retroreflective element. The retroreflective element angles represented by  $\alpha$  in Fig. 3 were from -45 degrees to 45 degrees every 15 degrees. The floating distance indicated by ZC in Fig. 3 was 200 mm, the distance from the beam splitter to the retroreflective element (ZR in Fig. 3) was 150 mm, and the in-plane angle ( $\beta$ ) of the retroreflective element was 45 degrees.

### 4.2.2 In-plane rotation of retro-reflective element

For investigating the change when the polarization rotation angle was changed significantly the MTF of the aerial image was measured by changing the in-plane rotation (angle shown by  $\beta$  in Fig. 3) clockwise from 0 degrees to 90 degrees. The initial position was set to 0 degrees, and ZC = 200 mm, ZR = 150 mm, and  $\alpha$  = 0 degrees in Fig. 3.

### 4.2.3 Floating distance

For investigating the change when the floating distance was changed, the MTF was measured by changing the floating distance shown by ZC in Fig. 3 of 100 mm, 200 mm, 300 mm, and 400 mm. ZR = 150 mm,  $\alpha$  = 0 degrees,  $\beta$  = 45 degrees in Fig. 3.

### 5 DEPENDENCE OF MTF ON 3 FACTORS OF RETRO-REFLECTIVE ELEMENT IN P-AIRR

**5.1 Angle of incidence on the retro-reflective element** The MTF dependence on incident angles are shown in Fig. 5. The overall MTF has a tendency of decreasing from 1.0 to around 0.0 as spatial resolution increased from 0.0 to 0.5 lp / mm.

The MTF decreases as the angle of incidence on the retro-reflective element increases from 0 to  $\pm$  45 degrees. At incident angles of  $\pm$  15 and  $\pm$  30 degrees, MTF decrease is small, but MTF decreases significantly at  $\pm$  45 degrees.

Symmetry was confirmed between the negative and positive directions of the incident angle, because the RF-Ax of a corner cube type retroreflective element has symmetry in the positive and negative directions of the incident angle. MTF decrease by incident angle increase is due to narrowing of the apparent opening as the incident angle increases and the spread by diffraction increase.



Fig. 5 MTF change due to angle of incidence.

#### 5.2 In-plane rotation of retro-reflective element

The MTF dependence on in-plane rotation of retroreflective element are shown in Fig. 6. Since the prism type retroreflective element has anisotropy [4], some fluctuations can be seen, but no significant difference can be seen between various in-plane rotation angles. This is caused by experimental setup that the retroreflective element with the retardation film attached was rotated. This leads to difficulty in separating the factors that cause the retroreflective element to rotate due to changes in the MTF.



Fig. 6 MTF change due to in-plane rotation of the retroreflective element.

### 5.3 Change the floating distance

The MTF dependence on floating distance are shown in Fig. 7. The overall MTF has a tendency of decreasing from 1.0 to around 0.0 as spatial resolution increased from 0.0 to 0.4 lp / mm.

The MTF quickly decreases as the floating distance increases. Between the floating distances of 100mm and 200 mm. MTF decrease is small, but MTF decreases significantly at 300 mm and 400 mm.

MTF decrease by floating distance increase is due to narrowing of the apparent opening as the floating distance increases and the spread by diffraction increase.



### **6 CONCLUSION**

The modulation transfer function (MTF) in the aerial imaging (p-AIRR) by retroreflection using polarization modulation was measured by changing the angle of incidence on the retro-reflective element, the angle of rotation of polarization, and the floating distance.

When the incident angle increased and the floating distance increased, the MTF decreased. The polarization rotation angle does not affect the MTF significantly.

As these dependence are almost the same as that in conventional AIRR without polarization modulation, both results indicate that the resolution of imaging in AIRR predominantly depends on the structure of the retroreflective element.

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