# Resolution Evaluation of Aerial Image Formed with AIRR by Use of Two Transparent Spheres 

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#### Abstract

This paper reports the modulation transfer function (MTF) for aerial imaging optical system with aerial imaging by retro-reflection (AIRR) by use of two transparent spheres. MTF is measured under different positions of the spheres. Experimental results suggest that placing transparent spheres in specific positions can increase MTF.


## 1 Introduction

Three-Dimensional (3D) displays are being researched and developed for various applications such as advertising, amusement, and medical equipment [1]. Aerial imaging by retro-reflection (AIRR) is one of the methods to realize 3D displays [2]. AIRR is a technique to display a real image visible to the naked eye in the mid-air.

In a previous study, a steganography of aerial images formed with AIRR with two transparent spheres was proposed [3]. This technique uses a transparent sphere as a key to conceal the aerial image, and then decodes the aerial image by placing another sphere of the same design appropriately. In addition, it has been shown that the transparent spheres can reduce the area of retro-reflector by acting as ball lenses. However, the effect of transparent spheres on imaging resolution has not been verified experimentally.

In this paper, we evaluate the resolution of aerial images formed by AIRR combining two transparent spheres in comparison with conventional AIRR. The modulation transfer function (MTF) can be measured by the knife-edge method [4,5]. The effect of the combination of transparent spheres on the resolution is examined in comparison with the aerial image formed by the conventional AIRR. In addition to the conventional setup, we also measure the resolution of the aerial image when the arrangement of the transparent spheres is changed in the optical path with plane symmetry. In general, the resolution of the aerial image is assumed not to be degraded by placing the spheres. However, when the transparent sphere is placed close to the light-emitting position, the transparent sphere may quasi-collimate the light and the formed aerial image may become less blur. Furthermore, the flexibility of moving the position of the transparent spheres in the optical path improves the flexibility of the configuration of the aerial image steganography.

## 2 Principles

### 2.1 Aerial Imaging Steganography by Use of Two Rransparent Spheres

Fig. 1 (a) shows the diagram of conventional AIRR. This setup consists of a light source, a beam splitter (BS), and a retro-reflector. The beam splitter reflects rays from the light source. The reflected rays are retro-reflected, that is, reflected reversely at the incident positions on the retro-reflector. The retro-reflected rays are converged to the plane-symmetrical position of the light source with respect to the beam splitter.

Fig. 1 (b) shows the diagram of our proposed method where the two same transparent spheres are placed plane-symmetrically respect to the beam splitter. The light emitted from the light source passes through Transparent sphere 1 with refraction (concealing), and then the light is reflected by the beam splitter and enters to the retro-reflector. The retro-reflected light passes through the beam splitter and is refracted by Transparent sphere 2 , which forms aerial image at the planesymmetrical position of the light source with respect to the beam splitter. Here, if there is no transparent sphere 2, then the aerial image cannot be formed (decoded). Also, if the transparent sphere 2 has a different shape than the transparent sphere 1, the same applies.


Fig. 1 Principle of aerial imaging. (a) Conventional AIRR; (b) AIRR with dual transparent spheres.

### 2.2 Slanted Knife-edge Method

MTF is one of the quantitative evaluation rate of imaging quality. The slanted knife edge method is a method to estimate the MTF curve of spatial frequency in one direction in the observation area based on the recorded image. In Fig. 2 (a), the observation area is extracted from the aerial image and converted to a grayscale image. In Fig. 2 (b), the edge spread function


Fig. 2 Knife edge method (a) extracting the observation area from the aerial image and converting it to a grayscale image; (b) projecting the oblique edge image.
(ESF) curve with an equivalently fine sampling interval is obtained by projecting the pixel values of the grayscale image onto the projection axis that is perpendicular to the edge and superimposing them. The noise in the ESP is removed by wavelet denoising, and the line spread function (LSF) is obtained by further differentiation. Furthermore, the MTF is derived by determining the absolute value using Fourier transform and normalization. The edge of this experiment is a slightly tilted knife edge that blocks the light source with the light intensity uniformed by the integrating sphere.

## 3 Experiments

Fig. 3 shows the optical system of this experiment. A halogen light source, HL-2000-LL, was used. The integrating sphere was an IS200-4 with an aperture diameter of 12.5 mm . Two glass transparent spheres with a radius of 30 mm were used. A glass plate was used for the beam splitter. A Nippon Carbide RF-Ax retro-reflector was used. The formed aerial images were captured by a digital camera [Nikon D5500].

Fig. 4 shows the parameters of variation in this experiment. Here, the light path from the light source changes depending on the arrangement of the transparent spheres. However, the imaging position of the aerial image does not change as long as the transparent spheres are placed symmetrically with respect to the beam splitter. First, as shown in Fig. 4 (a), the transparent sphere is moved left and right along the beam splitter, and the change in the aerial image caused by the gap between the center of the transparent spheres and the center of the light source is captured. Next, as shown in Fig. 4 (b), the change in the aerial image is captured when the transparent spheres are moved back and forward between the light source and the beam splitter. From the
measurement results under these conditions, we will confirm that aerial image steganography can be applied if the transparent spheres are arranged symmetrically between the light source and the beam splitter with a beam splitter.


Fig. 3 Experimental setup. (a) System layout; (b) photograph of the system.

(a)


Fig. 4 Variation parameters. (a) moving the sphere left and right along the beam splitter; (b) moving back and forward between the light source and the beam splitter.

In the actual experiment, light from the light source enters the integrating sphere, is reflected multiple times by the entire sphere, and is emitted as light with a uniform intensity distribution. The half-hidden half-moon shaped light emitted from the integrating sphere by the knife edge is refracted by the transparent sphere and enters the beam splitter, where it is reflected. The light reflected by the beam splitter is retroreflected by the retro-reflector, and follows a symmetrical refraction path on the transparent sphere arranged in plane symmetry to form an aerial image at the position of the knife edge and plane symmetry with the beam splitter as the axis.

The formed aerial images are recorded by a digital
camera under the following conditions: ISO: 400, fnumber: 4.5 , and focal length: 24 mm . A $100 \times 100 \mathrm{~mm}$ grayscale image is extracted from the recorded image, and the MTF curve is derived from it.

## 4 Results

### 4.1 Spheres Shift Parallel to the BS

Fig. 5 shows the aerial image of the slanted knife-edge method formed by AIRR when the transparent sphere was moved along the beam splitter. There was no visual difference in the blur caused by the shift between the center of the transparent sphere and the center of the light source. The same result can be obtained when compared to the case without the transparent sphere.

Fig. 6 shows the denoised ESF curve of the grayscale image in Fig. 5 projected onto the perpendicular projection axis. the ESF curve shown in Fig. 6 shows a decrease in the color shading of the aerial image when the transparent


Fig. 5 Aerial images and gray scale images when spheres were moved in parallel to the BS.


Fig. 6 ESF curve when spheres were moved in parallel to the BS.


Fig. 7 LSF curve when the spheres were moved in parallel to the BS.


Fig. $\mathbf{8}$ MTF curve when spheres were moved in parallel to the BS.
sphere is present compared to when it is absent. The ESF curve shown in Fig. 6 shows a decrease in the color shade of the aerial image when the transparent sphere is present compared to when it is absent, and the amount of blur is small between 40 and 60 pixels, regardless of the shift between the center of the transparent sphere and the center of the light source.

Fig. 7 shows the LSF curve derived by differentiating the noise-eliminated ESF curve in Fig. 6. The change in the curve is concentrated at the edges, indicating that there is little blur.

Fig. 8 shows that the MTF curve is almost constant regardless of the presence or absence of the transparent sphere, and regardless of the gap between the center of the transparent sphere and the center of the light source.

These graphs showed that the lateral shift of the transparent sphere does not affect the resolution of the aerial image.

### 4.2 Spheres Movement in Distance

Fig. 9 shows the aerial image formed when the transparent sphere is moved back and forward between the light source and the beam splitter, and the $100 \times 100$ grayscale image extracted from the aerial image. This photograph shows that the movement of the transparent sphere and the presence or absence of the sphere had no particular effect on the visual blur of the aerial image.

Fig. 10 (the ESF curve) shows a decrease in the color shade of the aerial image when the transparent sphere is present compared to when it is absent, similar to the results in 4.1.
Fig. 11 (the LSF curve) It can be seen that the change in pixel value is constant regardless of the presence or absence of the transparent sphere or the movement of the transparent sphere between the light source and the beam splitter. In addition, the change in the curve is concentrated at the edges, indicating that there is little blur.

Fig. 12 shows it was confirmed that the change in the slope of the MTF curve was almost constant regardless of the presence or absence of the transparent sphere or
the movement of the transparent sphere between the light source and the beam splitter.

It can also be found that the MTF is improved when the transparent sphere placed close to the integrating sphere. The reason why the MTF is improved only when the transparent sphere is reached to the integrating sphere is that in this arrangement, the light emitted from the integrating sphere becomes pseudo-parallel light due to the principle of the ball lens. Then, the wavefront incident on the retro-reflector becomes close to a plane wave, and the retro-reflector functions as an excellent pseudo-phase conjugate. At this time, the retro-reflected ray can follow the original path more faithfully due to the pseudo-phase conjugation characteristic. This less-lossy light is finally converged and formed by the transparent sphere, resulting in a reduction of blur.

## 5 Conclusions

From the results of the MTF curve obtained in this experiment, it was confirmed that the resolution of the aerial image was not affected by the presence or absence of a transparent sphere, the shift from the center of the light source, or the movement within the floating distance., except when the transparent sphere was in contact with the integrating sphere.

In addition, if we consider that the spread of the point image distribution function (the spread of rays) is based on the diffraction caused by the aperture of the retroreflector, the resolution does not depend on the presence or absence of a sphere.


Fig. 9 Aerial images and gray scale images of the spheres moved back and forward between the light source and the BS.


Fig. 10 ESF curve when the spheres moved back and forward between the light source and the BS.


Fig. 11 LSF curve when the spheres moved between the light source and the BS.


Fig. 12 MTF curve when the spheres moved between the light source and the BS.

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