Displays are Inherently 3dB Worse Than the Real Thing

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Keywords: Image acquisition, Display systems, Optical shot noise, Poisson distribution

ABSTRACT

The latest imaging technologies have successfully diminished two of the three significant camera noises (dark current noise and readout noise). However, optical shot noise still exists, which is inevitable by nature. This paper shows that even an ideal imaging system yields 3dB worse display quality than the real thing.

1 Introduction

We can imagine that in an ideal imaging system, the camera captures the brightness as it is, the signal is transmitted losslessly, and the monitor displays the signal as it is. Putting aside the limits of the spatial resolution, temporal resolution (frame rate), transmission delay, 3-dimensional information, and display intensity dynamic range, the reproduced image is imagined to be indistinguishable from the real thing. However, life is challenging. However, it is impossible with today's technology to achieve the ideal imaging, such as Fig. 1, because of (at least) the following reasons:

- Transmission delay
- Lack of frame rate
- Lack of spatial resolution
- Quantization of tones
- Insufficient brightness
- Insufficient darkness
- Lack of color gamut
- Convergence accommodation contradiction
- Lack of binocular disparity
- Lack of motion parallax

etc.

However, we may imagine that the advance of camera and display technologies will eventually achieve an indistinguishable playback system. The performance of image sensors has been increasing in recent years due to technological innovations. In CMOS image sensors, the reduction of dark current noise and readout noise, two of the three major noise factors in image sensors, has been affirmatively resolved [1], leaving optical shot noise as the only remaining noise factor. We have previously utilized this optical shot noise to enhance the image bit depth and measure light beyond the digital count limit brightness [2,3]. If all of the above reasons hinder an ideal imaging system, the question will be reduced to the following: "If a flat, stationary, matt object, that has the exact color and brightness the display can show is displayed, will it be indistinguishable from the real object?" (see Fig. 2).

This paper discusses what prevents the current system from displaying a brightness indistinguishable from the real object. We first explain a simple system that tries to reproduce high-fidelity images. Then we formulate the noise distribution contained in the captured signal and reproduced signal. The former obeys a well-known Poisson distribution, while the latter obeys a new type of probabilistic distribution which will be shown later. Consequently, the reproduced signal has twice as much noise variance as a captured signal.

Fig. 1 Ideal imaging system. Displayed image shall be indistinguishable with the real object behind.

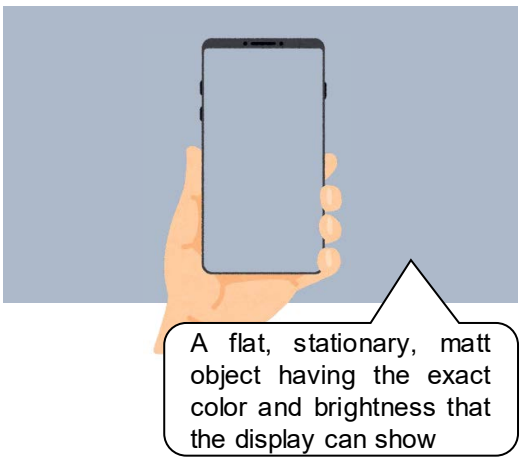

Fig. 2 Relaxed version of Fig. 1. We will investigate if the display and real object are indistinguishable.

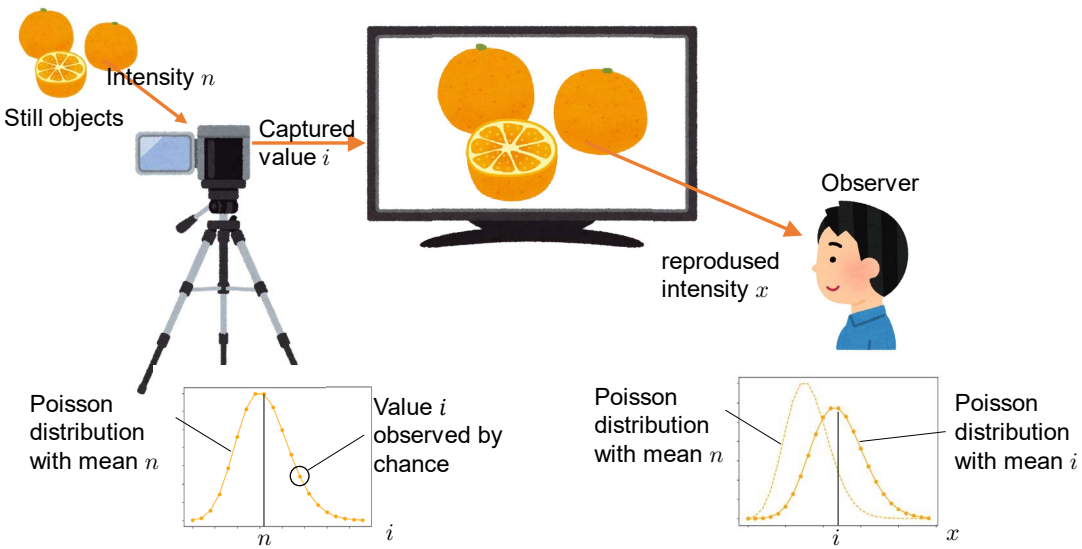

Fig. 3 Image acquisition, transmission, and display model with associated signal flow

2 Problem Formulation

2.1 Optical shot noise

Optical shot noise is caused by temporal fluctuations in the number of photons that come to the sensor as randomly as raindrops and cannot be eliminated in principle. Even a "seemingly" stationary light source emits photons randomly from time to time. The number of photons per certain period follows the Poisson distribution. One of the important properties of this distribution is that its mean and variance have the same value.

2.2 Problem Formulation

Fig. 3 shows the imaging system we consider. The objects are all stationary, and let us suppose a particular part of the object emits n photons per one frame period on average towards the camera. n can be a real number. An integer value i is the captured intensity value, i.e., the number of photoelectrons accumulated at one frame period (we assume the quantum efficiency of the image sensor to be 100%). Here i is an integer and a stochastic variable that obeys Poisson distribution with mean n (see the distribution graph in Fig. 2 in the bottom left). The display tries to emit light with the intensity of x, i.e., x photons on average per one frame period. Again, x is an integer and a stochastic variable that obeys Poisson distribution with mean i.

2.3 Meta-Poisson Distribution

The Poisson distribution is formulated as follows:

$$P_\lambda(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

where λ (a real value) is the mean (and also the variance) of the stochastic variable X. An integer x is the captured intensity. Based on this, the reproduced intensity x at the observer obeys the probability mass function (PMF) $f(n,x)$ is formulated as follows.

$$\begin{aligned} f(n,x) &= \sum_{i=0}^{\infty} P_{\lambda=n}(X=i)P_{\lambda=i}(X=x) \\ &= \sum_{i=0}^{\infty} \frac{e^{-n}n^i}{i!}\frac{e^{-i}i^x}{x!} \end{aligned}$$

This function satisfies PMF condition, i.e.,

$$\sum_{x=0}^{\infty} f(n,x) = 1.$$

Let us name this distribution "*meta-Poisson distribution*". Unfortunately, we do not have the closed form of this distribution (so far). However, we can calculate its mean to be n because

$$\begin{aligned} \sum_{x=0}^{\infty} f(n,x)x &= \sum_{x=0}^{\infty}\sum_{i=0}^{\infty} P_{\lambda=n}(X=i)P_{\lambda=i}(X=x)x \\ &= \sum_{i=0}^{\infty} P_{\lambda=n}(X=i)i \\ &= n. \end{aligned}$$

And its variance can be calculated as $2n$ because

$$\begin{aligned} &\sum_{x=0}^{\infty} f(n,x)(x-n)^2 \\ &= \sum_{x=0}^{\infty}\sum_{i=0}^{\infty} P_{\lambda=n}(X=i)P_{\lambda=i}(X=x)(x-n)^2 \\ &= \sum_{i=0}^{\infty} P_{\lambda=n}(X=i)\sum_{x=0}^{\infty} P_{\lambda=i}(X=x)(x-n)^2 \\ &= \sum_{i=0}^{\infty} P_{\lambda=n}(X=i)((i-n)^2+i) \\ &= 2n. \end{aligned}$$

This implies that the observer captures twice times of noise ($2n$) than the noise from the real thing (n), which means that the SNR of the reproduction image is theoretically (2x =) 3.0103 dB worse.

The graphs of Poisson and meta-Poisson distribution are depicted in Fig. 4.

3 Conclusion

In this paper, we have theoretically shown that captured and displayed images always have higher

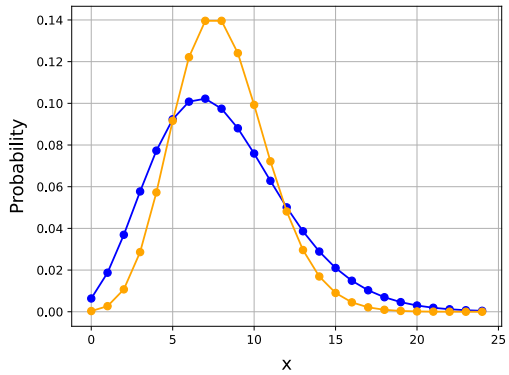

Fig. 4 Orange: Ordinary Poisson distribution $P_{\lambda=8}(X=x)$ (mean 8, variance 8), Blue: *meta-Poisson* distribution $f(8,x)$ (mean 8, variance 16)

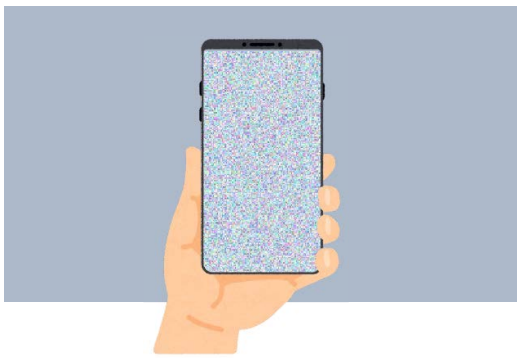

Fig. 5 The reality. Displayed image is always 3dB worse than the real thing.

noise than the real object, which obeys not ubiquitously existing Poisson distribution but a more flat, high-variance distribution. To our limited knowledge, it is first noted in this paper that any display systems inevitably have a 3dB worse reproduction quality than the real thing (see Fig. 5).

Obviously, this is the limitation of the current brightness-based camera & display model. Tackling this limitation is an exciting and challenging problem. One possible solution is to realize a photon-by-photon model like what mirror reflection is doing. It would be our future work to consider how to realize such a photon-based digital system.

Acknowledgement

This work was supported in part by the JSPS KAKENHI Grant Number JP 23K11155.

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