

# SUPER TWISTING CONTROL WITH BACKSTEPPING DESIGN FOR ELECTRO-HYDRAULIC SYSTEM WITH UNKNOWN PERTURBATION

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**Abstract.** In this paper, a robust control scheme is presented for controlling the position of the electrohydraulic servo system (EHSs). In the working environment and the EHSs, there always exist several perturbations such as temperature, internal and external leakages, uncertainty parameters and modeling errors; they affect the control performance of the system. So that, the robust control scheme with a first-order sliding mode control (SMC) for the mechanical dynamic and a super twisting control (STC) for the hydraulic dynamic based on the backstepping design are developed. It is called super twisting algorithm backstepping control (STABC). The STABC inherits the robust and stable properties of SMC, reducing chattering effects of (STC). The simulation results show that the proposed method guarantees the robustness and stability of the controlled system. Furthermore, it also suppresses the chattering effect more efficient than the backstepping sliding mode control.

Keywords: Backstepping, Super-Twisting Algorithm, adaptive control, Electro-Hydraulic system, uncertain parameters.

## INTRODUCTION

Electro-Hydraulic systems (EHSs) are widely used in industry due to the reliability, quick response, high torque to weight ratio, flexibility, and ruggedness. They are considered as potential choices for modern industries in areas ranging from heavy-duty manipulators to precision machine tools. However, the dynamics of the EHSs [1] are highly nonlinear, large uncertainties and working environments always contain some unknown perturbations, the control performance such as stability, frequency response, or loading sensitivity, will decrease or become worst. Since it is an important field of research, many different control strategies have been developed and applied to the EHSSs to improve the force/position control performance [2] [3] [4] [5] [6] [7] [8]. In [2], an adaptive backstepping control scheme was presented for position control of an electrohydraulic system with the uncertainties and a bounded disturbance. A fuzzy Proportional-Integral-Derivative (PID) control optimized by using a robust extended Kalman filter was presented in [3]. Based on the extended Kalman filter, the controller parameters were adjusted. In [4], An adaptive nonlinear controller was deployed for position tracking control of a Pump-controlled Electro Hydraulic System (EHS). The nonlinearities were compensated by an advanced backstepping based technique while all uncertainties were estimated directly from the state control error. A hydraulic load simulator for conducting performance and stability testing related to the form control problem of the electrohydraulic system was shown in [5]. A grey prediction model and a fuzzy PID controller were developed. The grey prediction compensator improved the system settle time and overshoot problems. In [6], the quantitative feedback theory was deployed to design a robust force control. The robust and tracking performance was guaranteed with the appearance of the disturbance. In [7], an adaptive sliding mode control was presented for an electro-hydraulic system with unknown nonlinear parameters. The nonlinear adaptive controller with adaptive laws compensated for the nonlinear uncertain parameters caused by the varieties of the original control volumes. A novel-type Lyapunov was developed to construct an asymptotically stable adaptive controller and adaptive laws. In [8] A nonlinear adaptive torque control was deployed for electro-hydraulic load simulator with external disturbance. The stability of the developed control algorithms was proven via Lyapunov analysis.

Sliding mode control (SMC) has been extensively deployed to control of uncertain nonlinear systems with the uncertain environment and external disturbances because of its robustness property [9]. The fundamental idea of the SMC is to use a discontinuous control term to drive the controlled system's error state variables to approaches to zero. Additionally, all state variable will track to the desired trajectory. However, the SMC presents a high-frequency switching control signal that leads to a chattering problem. The super-twisting algorithm (STA) [10] is one of the best solutions to reduce the chattering effects. The algorithm ensures the robustness respect to uncertainties, modeling errors and external disturbances while decreasing the chattering effect found in the conventional SMC algorithm. Recently, the STA has been demonstrated the stability based on the Lyapunov

approach [11] [12] [13]. In [11], the super-twisting algorithm, without or with perturbation, was provided. The Lyapunov function was used to demonstrate the stability and give an estimate of convergence time. A novel, Lyapunov-based, variable gain STA was proposed in [12]. It guarantees for the linear-time invariant system the global, finite-time convergence to the desired sliding surface when the matched perturbations/uncertainties and their derivative were the known functions. In [13], a novel super-twisting adaptive sliding mode control law was presented for control an electro-pneumatic actuator.

Additionally, the backstepping control is also a nonlinear control method which has widely been used to control the nonlinear system because of its systematic and recursive design method. When a control scheme is employed with the high order SMC based on the backstepping technique, it will inherit the robust characteristics [14], and make a simple way to apply and prove the robustness, and the stability of the controlled system [15]. In this paper, a robust control scheme, which consists of a first order sliding mode control, and a super twisting control, is proposed to control the position of the EHSs under the presence of the uncertainties based on the backstepping design. The first order SMC is used for the mechanical control, and the STC is utilized for the hydraulic control. This combination will reduce the order control, remedy the chattering effects, and give a systematic, and recursive method. To validate the effectiveness, and chattering remedy of the proposed STABC, some simulations are carried out, and these results are compared with the backstepping sliding mode control which used two first order SMC to control the EHSs, and it is also designed based on the backstepping technique.

This paper is organized as follows. Section 2 gives the dynamic model of the electro-hydraulic system. Section 3 describes the proposed control. Then Section 4 derives the simulation results. Finally, the conclusions are presented in Section 5.

#### **DYNAMIC MODEL**

A schematic of the electrohydraulic system used in this paper is shown in FIGURE 1. The system includes a servo valve, a rotary hydraulic actuator, two pressure sensors, a position sensor, and a hydraulic power unit.



FIGURE 1. Schematic of the electrohydraulic system

#### **Mechanical Subsystem Dynamic**

From FIGURE 1, the dynamic of the inertial load can be described by:

$$I\ddot{\theta} = \tau_L - B\dot{\theta} + \xi_1(t) \tag{1}$$

where  $\theta$  and *I* denotes the angular displacement and the inertial mass of the load, respectively;  $P_L = P_1 - P_2$  is the load pressure of the hydraulic actuator,  $P_1$  and  $P_2$  are the pressures inside the two chambers of the actuator; *A* is the radian displacement of the actuator; *B* is the coefficient of the viscous friction; and  $\xi_1(t)$  represents other disturbances like the un-model nonlinear frictions as well as external disturbances.

#### Hydraulic Subsystem Dynamic

As shown in FIGURE 1, the electro-hydraulic actuator consists of the servo-valve and a rotary actuator. With the servo-valve, the valve opening  $x_v$  is proportional to input current/voltage, so  $x_v$  can be considered as actuator's input. By supplying pressure  $P_s$  and the spool displacement  $x_v$ , the servo-valve can provide the chamber pressure  $P_1$ ,  $P_2$ , and the flow-rate  $Q_1$ ,  $Q_2$ , respectively. Considering the leakage and the compressibility, the dynamics are presented by

$$\dot{P}_{1} = \frac{\beta}{V_{01} + A\theta} \left( Q_{1} - A\dot{\theta} - Q_{LI} \right)$$
<sup>(2)</sup>

$$\dot{P}_2 = \frac{\beta}{V_{02} - A\theta} \left( -Q_2 + A\dot{\theta} + Q_{LI} \right)$$
(3)

where  $V_{0i}$  and A show the volume and radian area of the two sides of the rotary actuator respectively,  $\beta$  is the bulk modulus of the fluid,  $Q_{LI}$  is the internal leakages.

Assumption 1: The internal leakage is a bounded uncertainty, and it can be presented as follows:

(4)

The fluid flow rates in the chambers of the rotary actuator  $Q_1$ ,  $Q_2$  are nonlinear functions of the spool valve displacement and the pressure. The functions can be expressed as

 $|Q_{II}| \leq \delta_0$ 

$$Q_{1} = \begin{cases} C_{d} w x_{v} \sqrt{\frac{2}{\rho} (P_{s} - P_{1}), x_{v} \ge 0} \\ C_{d} w x_{v} \sqrt{\frac{2}{\rho} (P_{1} - P_{r}), x_{v} < 0} \end{cases}; \qquad Q_{2} = \begin{cases} C_{d} w x_{v} \sqrt{\frac{2}{\rho} (P_{2} - P_{r}), x_{v} \ge 0} \\ C_{d} w x_{v} \sqrt{\frac{2}{\rho} (P_{s} - P_{2}), x_{v} < 0} \end{cases}$$

Where  $C_d$  presents the discharge coefficient;  $\omega$  is the area gradient of the servo-valve;  $\rho$  is the fluid density,  $P_r$  is the pressure of the tank.

The torque generated by the rotary actuator can be calculated as

$$\tau_L = A \left( P_1 - P_2 \right) \tag{5}$$

Take time derivative of the torque equation; we can obtain the dynamics for the rotary actuator as

$$\dot{\tau}_{L} = A \left( \dot{P}_{1} - \dot{P}_{2} \right) = If \left( \theta, \dot{\theta} \right) + Ig \left( \theta, P_{1}, P_{2} \right) x_{v} + Q_{LI}$$

$$(6)$$

where 
$$f(\theta, \dot{\theta}) = -\frac{\dot{\theta}\beta A^2}{I} \left[ \frac{1}{V_{10} + A\theta} + \frac{1}{V_{20} - A\theta} \right]; g(\theta, P_1, P_2) = \frac{\beta A}{I} \left[ \frac{Q_1}{V_{10} + A\theta} + \frac{Q_2}{V_{20} - A\theta} \right]$$

As the electro-hydraulic servo-valve is considered in this work and the valve opening  $x_v$  is proportional to the input signal u. Then, u can be replaced  $x_v$  in the model

$$\dot{\tau}_{L} = If\left(\theta, \dot{\theta}\right) + Ig\left(\theta, P_{1}, P_{2}\right)u + Q_{LI}$$

$$(7)$$

The state variables of the system are defined as  $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} \theta & \dot{\theta} & \frac{\tau_L}{I} \end{bmatrix}^T$ . Then, the system can be

represented in a state space

$$x_{1} = x_{2}$$

$$\dot{x}_{2} = x_{3} + d_{1}(t)$$

$$\dot{x}_{3} = g(x)u + f(x) + d_{2}(t)$$
(8)

where  $d_1(t) = \frac{\xi_1(t)}{I}$ ,  $d_2 = \Delta f + \frac{Q_{LI}}{I}$ , and

#### **CONTROL DESIGN**



FIGURE 2 The structure of the proposed control

In this research, a robust control via the backstepping approach [16] and sliding mode control [10] aims to control position for the EHSs. The structure of the proposed control is shown in FIGURE 2. It consists of two control

loops; the outer loop is the position control which is designed by sliding mode control; the inner loop is the torque control which is derived by super twisting control.

Assumption 1: The uncertainty,  $d_1$ , in the mechanical dynamic is bounded and is expressed as follows:

$$\|d_1\| \le \delta_0 \tag{9}$$

Assumption 2: The uncertainty,  $d_2$ , in the hydraulic dynamic is bounded and is given as follows:

$$\begin{aligned} u_{2} &= u_{21} + u_{22} \\ \left| d_{21} \right| &\leq \delta_{1} \left| s_{2} \right|^{1/2} \\ \left| \dot{d}_{22} \right| &\leq \delta_{2} \end{aligned}$$
(10)

**Step 1:** The sliding mode control for the mechanical dynamic is investigated to ensure the position response tracking to the desired position.

Define state variable errors:

$$e = x_1 - x_{1d} \in R^2 
\dot{e} = \dot{x}_1 - x_{2d} \in R^2$$
(11)

The sliding variable is chosen as follows:

$$s_1 = \dot{e} + ce \tag{12}$$

where c is a positive-definite constant Set the torque virtual control error as follows:

$$s_2 = x_3 - x_{3d} \tag{13}$$

The desired torque is chosen as follows:

$$x_{3d} = -k_1 s_1 + \dot{x}_{2d} + c\dot{e} + \eta_1 sign(s_1)$$
(14)

where  $k_1$  is the positive definite constant,  $\eta_1$  is the positive definite robust gain; it is selected how to  $\eta_1 \ge \delta_0$ **Step 2:** The super twisting control [13] for the hydraulic dynamics is designed to guarantee the torque error as small as possible.

The control effort is chosen as follows:

$$u = u_{eq} + u_{STA} \tag{15}$$

with

$$u_{eq} = g^{-1} \left( -\frac{1}{\lambda + 4\varepsilon^2 - 2\varepsilon} s_1 \left| s_2 \right| - f + \dot{x}_{3d} \right)$$
  

$$u_{STA} = g^{-1} \left( -\eta_2 \left| s_2 \right|^{1/2} sign(s_2) + v \right)$$
  

$$\dot{v} = -\eta_3 sign(s_2)$$
(16)

where  $\eta_1$ , and  $\eta_2$  is the robust gain of the hydraulic actuator, and they must satisfy the conditions as follows:

$$\eta_{2} > \frac{\delta_{1} \left( \lambda + 4\varepsilon^{2} \right) - \varepsilon \left( 4\delta_{2} + 1 \right)}{\lambda} + \frac{\left[ 2\varepsilon\delta_{1} - 2\delta_{2} - \lambda - 4\varepsilon^{2} \right]^{2}}{12\varepsilon\lambda}$$
$$\eta_{3} = 2\varepsilon\eta_{2}$$

where  $\varepsilon$ ,  $\lambda$  are arbitrary positive constants.

### SIMULATION EVALUATIONS

To explain aforementioned designs and fundamental problems associated with high-accuracy tracking control of electro-hydraulic rotary actuators, a simulation has been set up with the presence of internal leakage and friction. The platform consists of a bench case, hydraulic position system, including a hydraulic actuator, a rotary encoder whose accuracy is about 10", two pressure sensors whose accuracy is 1 (bar), a servo valve whose bandwidth is above 150(Hz), a shaft joint, and a set of inertial steel sheets, etc., a hydraulic supplement and a measurement and control system. The sample time is 0.1ms. The parameters of the simulation system are shown as A =  $1.2x10^4$  m<sup>3</sup>/rad; I=0.327 Kg.m<sup>2</sup>; B=40 Nms/rad; V<sub>01</sub>= $1.15x10^{-4}$  m<sup>3</sup>; V<sub>02</sub>= $1.15x10^{-4}$  m<sup>3</sup>; Ps=100 Bar; Pr=3 Bar; k<sub>t</sub>= $2.16x10^{-8}$  M<sup>3</sup>/s/V/Pa<sup>1/2</sup>;  $\beta e=1.25x10^9$  Pa; C<sub>1</sub>= $-7.6952x10^{-20}$  M<sup>3</sup>/s/Pa/Pa; C<sub>2</sub>= $2.7594x10^{-12}$  M<sup>3</sup>/s/Pa; C<sub>3</sub>= $-0.2x10^{-5}$  M<sup>3</sup>/s.

The proposed control is compared to a conventional backstepping sliding mode control (BSMC) to validate the effectiveness of the proposed control. The BSMC have a same design procedure as the proposed control, but the super twisting control in the hydraulic dynamic is replaced by a first-order sliding mode control. Additionally, the perturbation,  $d_1(t)$ , in the mechanical dynamic, presents the friction, and it is derived as follows:

$$d_1(t) = \begin{cases} 50 & x_2 \ge 0\\ -50 & x_2 < 0 \end{cases}$$
(17)

The perturbation,  $d_2(t)$ , in the hydraulic subsystem dynamic, shows the internal leakage, and it is obtained from the following equation:

$$d_{2}(t) = \frac{Q_{LI}}{I} + \Delta f$$

$$Q_{LI} = c_{1}P_{L}^{2} + c_{2}P_{2} + c_{3}$$
(18)

The simulations are performed on Matlab2015a/Simulink which is equipped with a computer whose configuration is Intel® core i5-2500 CPU 3.30GHz, NVIDIA GeForce 440, RAM 6GB. The parameters of the controllers are shown as BSMC:  $\eta_1 = 70; c_1 = 200; k_1 = 200; \eta_2 = 5.10^{10}$ ; STABC:  $\eta_1 = 70; c_1 = 200; \eta_2 = 10^9; \eta_3 = 5.10^7; \varepsilon = 0.025; \lambda = 1.0475$ 

The reference signal,  $x_{1d}(t) = 17 \sin(10t) \binom{o}{}$ , is used for controlling the position and the simulations are carried out in 10 seconds. To demonstrate the stability, the robustness and the chattering suppression of the proposed control, two simulation cases are considered, where case 1 denotes the uncertainty of the hydraulic dynamic,  $\Delta f$ , in the first five seconds, and the uncertainty in the last five seconds are considered in case 2. The uncertainty is defined as follows:

$$\Delta f = \begin{cases} +0.5f(t) & 0 \le t \le 5\\ -0.5f(t) & 5 < t \le 10 \end{cases}$$
(19)



FIGURE 3 The performances of the BSMC and STABC with (a) the control performances, (b) the errors



FIGURE 4. The torque responses of (a) the BSMC, and (b) the STABC

The control performances and errors of the BSMC and STABC are depicted in FIGURE 3. As shown in FIGURE 3b, the tracking responses are influenced by the uncertainties and two robust controls are guaranteed that the errors are bounded by  $0.05^{\circ}$ . However, the BSMC with the discontinuous term of the robust control directly affects the torque performance and causes the chattering control signal as shown in FIGURE 4a, and FIGURE 5a, respectively.

To reduce the chattering effects, a super twisting algorithm in the proposed STABC is used to replace the first order sliding mode control which regulates the torque in the BSMC. This proposed control remedied the chattering in the torque control and the control signal as presented in FIGURE 4b, and FIGURE 5b, respectively.

#### CONCLUSION

This study has successfully carried out a STABC for controlling the position of an electrohydraulic actuator under the uncertainties such as modeling error, friction, and leakage. The STABC inherited with properties of the backstepping control, sliding mode control and super twisting algorithm possesses robust control characteristics, simple design, reducing chattering effects. Additionally, some simulation results were implemented and were compared with the BSMC to validate these robust properties and chattering reduction of the proposed control.



FIGURE 5. The control signal of (a) the BSMC, and (b) the STABC

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