The numerical model of natural hazards development in the environment stressed by opposing forces

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Natural hazards include earthquakes, tsunami, volcanic eruptions, floods, etc. The time of appearance of such significant events within hundreds of years can be considered as random. In most cases, the dangers’ amplitudes are not amenable to prediction, i.e. their size is also random. From the mathematical point of view, the deposition of natural hazards is described by exponential dependence, which is connected with the involvement of the own “mass” of danger. In the presence of opposing forces in a first approximation, these processes are described by the Verhulst equation. It is a particular variant \((Q<0, A=L)\) of the total autonomous differential equations of the 2nd order for the function \(x(t)\) on time \(t\), i.e. \(\frac{dx}{dt}=N+L\ast x+Q\ast x\ast x\), where \(N, L, Q\) are constants of equation with initial conditions \(t(n)\) and \(x(n)\).

The complete solution of this equation with arbitrary initial conditions has bulky appearance, although the logistic curve reflects it qualitatively quite well. However, these solutions allow us to reveal a violation of the principle of stability of numerical solutions of the logistic equation \(x(n+1)=x(n)+(1+a\ast (1-x(n)))\), where \(a=A\ast ((t(n+1)-t(n))\), when the derivative \(\frac{dx}{dt}\) is replaced by the value for \((x(n+1)-x(n))/(t(n+1)-t(n))\).

It is shown that the instability of the processes with the opposing factors invoked by jumps of initial conditions on consecutive segments. For certain values of the parameters of the differential equation associated with capacity of the stressed environment, both volatile and deterministic modes of development of the variable \(x(t)\), normalized to unit, can be formed. An example of the Verhulst model with parameter \(A\) shows the dependence of the solutions \(x(t)\) at time intervals \((t(n+1)-t(n))\) and tabular values of \(x(n)\) and different \(a\) jumps of initial conditions. Negative inclinations of dependency associated with the tabular values \(x(n)\) are shown. Thus, there appears a situation, which leads to the release of the variable \(x\) from the corridor, normalized per unit, of sustainable values. For each \(a\)-case, the changes in the structure of \(x(t)\) in time look diverse and complex.

Therefore, the numerical logistic equation can be taken as a numerical model for the development of natural hazards in the geographical environment, characterized by capacity (option a) of a tension of opposing factors.

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