

Uncertainty Estimate in Runoff Analysis Based on Theory of Stochastic Process

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The aim of this paper is to provide a theoretical framework of uncertainty estimate on rainfall-runoff analysis based on theory of stochastic process. In this study, we applied SDE (stochastic differential equation) and Fokker-Planck equation to estimate uncertainty of rainfall-runoff analysis. SDE (stochastic differential equation) based on this theory has been widely used in the field of mathematical finance due to predict stock price movement. Meanwhile, some researchers in the field of civil engineering have investigated by using this knowledge about SDE (stochastic differential equation) (e.g. Kurino et.al, 1999; Higashino and Kanda, 2001). However, in the field of civil engineering, there have been no researches about uncertainty estimate by using correspondence relationship of SDE (stochastic differential equation) and Fokker-Planck equation. The Fokker-Planck equation is a partial differential equation that describes the temporal variation of PDF (probability density function), and there is evidence to suggest that SDEs and Fokker-Planck equations are equivalent mathematically.

In this paper, therefore, the uncertainty of discharge on the uncertainty of rainfall is explained theoretically and mathematically by introduction of theory of stochastic process. The rainfall data is generally used as input data for rainfall-runoff analysis. The lumped rainfall-runoff model is represented by SDE (stochastic differential equation) due to describe it as difference formula, because the temporal variation of rainfall is expressed by its average plus deviation, which is approximated by Gaussian distribution. This is attributed to the observed rainfall by rain-gauge station and radar rain-gauge system. Moreover, we obtained the time evolution of PDF (probability density function) of water level or discharge from correspondence relationship of SDE (stochastic differential equation) and Fokker-Planck equation.

As a result, this paper has shown that it is possible to evaluate the uncertainty of discharge by using the relationship between SDE (stochastic differential equation) and Fokker-Planck equation. Moreover, the results of this study show that the uncertainty of discharge increases as rainfall intensity rises and non-linearity about resistance grows strong. These results are clarified by PDFs (probability density function) that satisfy Fokker-Planck equation about discharge. It means the reasonable discharge can be estimated based on the theory of stochastic processes, and it can be applied to the probabilistic risk of flood management.

Keywords: uncertainty, runoff analysis, lumped model, SDE, Fokker-Planck equation