The effective PML absorbing boundary condition for linear long-wave and linear dispersive wave tsunami simulations

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Tsunami simulations in regional scale usually performed in the bounded domain with appropriate absorbing boundary condition surrounding the computational area, for avoiding fictitious reflections from the model boundary contaminates the simulated tsunami wavefield. For such purpose, the Sommerfeld or the sponge boundary conditions are widely used. In the present study, we report that the new Perfectly Matched Layer condition, originally proposed in electromagnetics and being used widely in earthquake seismology, applied to the tsunami numerical simulation problem gives significant improvement on the quality of the boundary condition.
The PML is a sort of the sponge boundary condition, which damps outgoing tsunami waves by absorbing layer placed surrounding the numerical model with a finite thickness. In this PML region, physical variables were decomposed into directions according to the directions of their derivatives. Then, only a wave propagating perpendicular to the model boundary is absorbed so as to avoid artificial reflection from the boundary. A wave propagating parallel to the boundary is unchanged. This decomposition is a key to provide high-quality on absorbing outgoing waves without fictitious reflection. The linear-long wave tsunami perfectly suite the PML condition. In the case of the linear dispersive tsunami, however, the decomposition is not straightforward because the momentum equation contains a term of higher-order derivatives along the mixed directions. Therefore, we introduced a weighting factor in the PML absorber region, so that the effect of tsunami dispersion is gradually decreased as the tsunami penetrates towards the absorber region. Under this assumption, tsunami equation approaches to the linear long-wave equation, which enable us to utilize the PML equation.
To examine the efficiency of the proposed PML absorbing boundary conditions for numerical simulation of a tsunami, we performed a finite difference simulation tests under smooth and realistic bathymetry models using the PML condition. For comparison, we also simulated the tsunami with the use of the Sommerfeld radiation condition and a traditional sponge condition. In addition, we performed a simulation with a larger, boundary-free model for the reference. In both cases, the tsunami simulation with PML boundary condition shows significantly improved results having little fictitious reflection from boundaries. The Sommerfeld radiation condition in particular shows degraded performance for strongly dispersive waves. This is due to the mismatch between the dispersed various tsunami velocity and long-wave tsunami velocity assumed in the condition. The sponge boundary shows moderate performance, however it tends to have strong fictitious reflection for near-parallel angle incidence to the boundary. The PML boundary was always superior, even for the approximate implementation used in the linear dispersive waves.
In the regional tsunami modeling, the absorbing boundary is usually set at offshore where the water is deep. Therefore, the fictitious reflection at the boundary quickly come back to the model area with increasing amplitude as the water gets shallower, and it easily contaminate the model area. Because the computational loads for linear long-wave tsunami simulation is not so heavy for recent computers, one might extend the model area to avoid such reflection. However, the same procedure for the linear dispersive waves is not realistic, because its computational cost is 50-100 times as high as that for the linear long wave tsunami. The proposed PML boundary condition significantly improve the quality of tsunami numerical modeling without increasing computational cost.

Keywords: Tsunami, Numerical simulation, Finite difference method, Absorbing boundary condition, PML

