A new rheological model of magma for representing transigion from flow to fracture

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Whether a flowing magma with increasing strain rate becomes brittle or ductile is an important but puzzling problem in considering eruption dynamics. The criterion of the brittle/ductile transition that is currently used in volcanology is thoroughly based on the linear viscoelastic model assuming small deformation. However, flow of magma to fracture is not in the linear regime, and the current model to describe the phenomenon reveals inconsistency. Here the problem is re-considered and a new constitutive equation for magma is proposed.
Magma rheology is frequently represented by the linear Maxwell model, which consists of a viscous element and an elastic element in line. It behaves elastically in a short time or in high-frequency oscillation, and viscously in a long time or in low-frequency oscillation. The dynamic viscosity is defined in oscillatory deformation as the amplitude ratio of the stress to the strain rate and is a function of frequency. For both of magma and the Maxwel model, the dynamic viscosity is constant in the low-frequency viscous regime, and decreases as frequency increases in the elastic regime. The frequency separating the viscous and elastic regimes is inversely proportional to the low-frequency limit of the dynamic viscosity.
There are two well-known rheological laws that generally hold for polymeric fluids including magma. The one is time-temperature superposition, which indicates that change of rheology with decreasing temperature is equivalent with increasing frequency. The other is what is called Cox and Merz rule: the steady-state viscosity as a function of strain rate under continuous flow is the same as the dynamic viscosity as a function of frequency and it decreases with increasing strain rate.
Combining the two laws has lead the idea that flowing magma may enter the glassy (elastic) regime either by cooling or increasing strain rate (Dingwell, 1996). This idea has significantly influenced modeling of volcanic phenomena. Although the viscosity increases with cooling, it decreases with increasing strain rate. It is not clear whether transition to the glassy state with decreasing viscosity by increasing strain rate is the same as that by cooling.
In the area of non-linear physics, on the other hand, the glassy state is regarded as the jammed state and the glassy to ductile transition is linked to yielding (Trappe et al., 2002). In this view, a material goes from a glassy state to flow with increasing strain rate, which is opposite to the concept in volcanology. Moreover, Miyazaki et al. (2006) shows that the dynamic viscosity in the oscillatory flow either decreases or increases as the strain rate amplitude increases depending on how it is increased, that is the product of the frequency and the strain amplitude is increased whether by increasing the frequency or by increasing the strain amplitude.
The linear Maxwell model does not represent the Cox and Merz rule or non-linear behavior in large deformation. Here a new phenomenological model is proposed, which consists of the Maxwell model with a viscous element having variable viscosity and an equation representing change of the viscosity by forcing and relaxation. It can represent the behavior similar to the Cox and Merz rule. Using the new model, it is shown that the not the strain rate but the strain acceleration is essential for bringing the flow to the brittle regime.

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