

A Source Inversion Method with Realistic Error Model

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Use of a proper likelihood function through incorporation of modeling error and a realistic noise model are essential part of a source inversion analysis, because the shape of the likelihood function affects choice of hyperparameters, the maximum a posteriori (MAP) estimate and its uncertainty estimate. We propose an empirical Bayes method for kinematic linear source inversion with physically based modeling error and realistic noise covariance.

The colored noise effects have been incorporated into analyses of Interferometric Synthetic Aperture Radar (InSAR) and global navigation satellite system (GNSS) data. Recently, effects of colored noise for centroid moment tensor (CMT) inversion were also discussed. However, the colored noise effects were usually ignored in source inversion analyses. In the proposed method, a noise covariance matrix is constructed from continuous records before P arrivals and uncertainty of phase picking.

In earlier studies, both amplitude of noise and a weight of a priori information were treated as hyperparameters. In the proposed method, we reformulated the marginal likelihood function to use the known noise covariance matrix estimated from data before P arrivals and phase picking errors. As we are not able to know the true Earth structure, the calculated Green's functions contain modeling error, and incorporation of the modeling error is unavoidable for the source inversion analysis. Preceding studies approximated effects of modeling error by additional multivariate Gaussian noise (model noise) for data. One of the advantages of the previous approach is its simplicity. As a posterior probability distribution should still be a multivariate normal distribution, MAP estimation and its uncertainty estimation are straightforward. However, even when assuming multivariate Gaussian error for the elements in the coefficient matrix, it is shown that the theoretical likelihood function is a skewed function and not a multivariate normal distribution function. Thus, the previous approach biases the MAP estimate and potentially affect choice of hyperparameters. We propose another approach, which does not use model noise approximation, to incorporate effects of modeling error into source inversion analysis. In the present approach, the Earth structure is assumed to be a random variable, which follows a known probability distribution. Then, the Earth structure is marginalized to obtain a posterior probability distribution of the source process. The proposed approach naturally incorporates associations of modeling errors for different type of data (e.g. seismic waveforms and surface displacements). As the marginalization is not analytically possible in most cases, we use a Monte-Carlo method and obtain the posterior probability distribution as a finite mixture of multivariate normal distributions. The MAP estimate is obtained by using a numerical optimization technique.

Keywords: Source inversion, Empirical Bayes method, Modeling error