The difference between JMA magnitude and moment magnitude in terms of seismic efficiency

*Kiyohiko Yamamoto

1. Introduction: For large earthquakes occurring along the Japan trench except for the off Miyagi Pref. region, moment magnitude Mw is 0.4 larger than the JMA magnitude Mj. Here, Mj and Mw are proportional to logarithms of seismic energy Es and released moment Mo, respectively. Mo is proportional to the relative displacement ub of fault surfaces. Es depends on directly seismic efficiency f, but Mo does indirectly. Thus the difference is thought to reflect the degree of the dependence on f.
f is a function of the rupture velocity Vr and is large for a large Vr. The small Mj compared with Mw thus suggests the small Vr. The difference between Mj and Mw is discussed from this viewpoint for The Tohoku earthquake (2011/3/11, Mw9) as an example.
2.Theory: The damage zone fault model of earthquakes* is employed for the present discussion. In this model, a fault zone with a uniform thickness constitutes of damaged rock area and asperity area. Fault surfaces mean the boundaries between fault zone and host rock blocks. The damaged rocks have relaxed during a long time after the preceding faulting. An asperity has the same elastic constants as the host rocks. Faulting occurs at the time that the relative displacement ub reaches the critical value.
For faulting, energy balance is written by
Pa+Pb=Es+W, Es=f x Pb. (1)
Here, Pa and Pb respectively are strain energies in the asperity and in the host rock blocks. Pb is approximated by the strain energy released when a circular crack is produced in a homogeneous host rocks under the uniform stress, that is equal in magnitude to the average stress drop due to faulting.
W is apparent fracture energy that is equivalent to the work to the host rocks done by the vertical displacement of the fault surfaces. The displacement is produced by the rotation of damaged rocks accompanied by the rupture propagation in a fault zone.
A linear relation has been found between the width of fault damage zone and the length of fault (Vermilye, J. M., and C. H. Scholz, 1998). In order to link the model to fault size, the linear relation is adopted.* Further, Sato and Hirasawa (1973) present an approximate relationship between Vr and f for a circular crack. This relationship is used for the present discussion.
3. Results: For f=1, all strain energy Pb is dissipated as Es. The fraction of asperity area is about 2% of the fault zone area. Vr is approximately equal to the S-wave velocity of host rocks. For f close to zero, all strain energy Pa+Pb is used for the rupture propagation and Es and Vr go to zeros. These may be the characteristics of so-called slow slip events. The fraction tends to about 0.74%. This is 0.37 times of the fraction at f=1. This means that the displacement and the average stress drop decrease to 0.37 times of those at f=1.
The relationships between Es and Mj and between Mo and Mw respectively are written by
logEs = 1.5M + 4.8 (2)
LogMo = 1.5Mw + 9.1. (3)
For a constant fault area, Eq. (2) and Eq. (3) intersect around f = 0.8. This suggests that the seismic efficiency is about 0.8 for majority of earthquakes. For f = 0.8, Vr is determined at about 0.8 times of S-wave velocity Vs of host rock.
Referring to the report by JMA**, Mj and Mw of The Tohoku earthquake are 8.4 and 9.0, respectively, and Vr and Vs are about 1.8km/s and about 3.4 km/s, respectively. Vr is about 0.53 times of Vs. From the relationship of Vr and f, f is estimated at about 0.3. Mj is estimated at about 8.6 for f

= 0.3 and Mw=9.0. The estimated Mj tends to the observed one. This suggests that the small Mj is due to the small Vr.

Note: *Yamamoto and Yabe, 2009; http://kynmt.in.coocan.jp/ ;(REFERENCE/23)
**http://www.jma.go.jp/jma/kishou/books/gizyutu/133/ALL.pdf

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