Theoretical calculation of internal stress/strain changes caused by earthquakes: the effectiveness of the reciprocity theorem in a spherical earth

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Co-seismic deformation (e.g. stress and strain changes) within the Earth has been estimated by using Okada's (1992) formulae for a semi-infinite homogeneous medium. An example is seen in estimation of Coulomb's static stress changes. However, when stress or strain changes caused by great earthquakes are estimated, we should use a more realistic earth model including the effects of Earth's curvature and layered structure; the stress changes caused by the 2011 Tohoku-oki earthquake exceed 0.1 bar even at the epicentral distance over 400 km (Toda et al., 2011). In principle, Takeuchi & Saito (1972) showed a recipe to calculate deformations due to earthquakes in a spherical earth model. In practice, however, there are some computational problems in order to realize the computation of internal deformations. One of them is loss of significant digits (LSD). We found that a method using the reciprocity theorem (Okubo, 1993) can avoid this problem. In this presentation, we will show that the conventional method cannot avoid the LSD whereas the method using reciprocity theorem can.
After Takeuchi & Saito (1972), the equations governing co-seismic deformation field result in first-order inhomogeneous differential equations by expanding displacement and stress by the spherical harmonics. To solve these differential equations, in the conventional method, (i) we obtain the complementary solutions, (ii) find the particular solution, and (iii) add them so that the final solution satisfies the surface boundary condition. The magnitude of the final solution is the order of $(r_s/r_p)^n$, where the rs and r_p ($>r_s$) are the radii where the source is located and the deformation is evaluated, respectively, and n is the degree of the spherical harmonics. On the other hand, the magnitude of the solutions obtained by the process (i) and (ii) is the order of $(r_p/r_s)^n$. This means that, in process (iii), LSD cannot be avoided at a large degree n because we need to add the numbers whose magnitude is $(r_p/r_s)^n/(r_s/r_p)^n=(r_p/r_s)^{2n}$ times larger than that of the final solution. For example, the ratio $(r_p/r_s)^{2n}$ becomes 10^{12} at n=8,000 when the deformation at a depth of 10 km (r_p=6361 km) due to a source at a depth of 20 km (r_s=6351 km) is considered. We confirmed that LSD occurs around n=8,000 in actual computation.
In the method using the reciprocity theorem, we obtain (a) the solutions x_1 at the radius r_s caused by external sources such as tide and (b) a solution x_2 that has a unit jump at the radius r_p. They are easily calculated by numerical integration. (c) Finally, the final solution is obtained by multiplying x_1 and x_2 together. This means that we can avoid the LSD that occurs in doing addition.

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