A dynamical system of conduit flow with magma density change due to gas escape

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In lava dome eruptions, magma viscosity change due to crystallization and magma density change due to gas escape during magma ascent generate positive-feedback mechanisms in conduit flow: as magma discharge rate increases, effective magma viscosity decreases because of delay of crystallization (i.e., reduced viscous wall friction) and magma density increases because of less efficient gas escape (i.e., reduced gravitational load), leading to further increase in the discharge rate. These feedback mechanisms induce complex features of conduit flow such as a cyclic behavior and a drastic change in flow pattern. The effect of magma viscosity change on conduit flow has already been investigated in detail in previous studies based on the modeling of a dynamical system of conduit flow. On the other hand, the effect of magma density change is not well understood, although the importance of this effect has been implied from numerical results of 1-dimensional conduit flow model. In this study, we developed a model for a dynamical system of conduit flow in which magma density change due to gas escape is taken into account, and investigated the effects of the magma density change on conduit flow dynamics.

In our model, flow variables in a cylindrical conduit are spatially averaged in vertical direction, and the conduit is connected with magma chamber surrounded by elastic rocks. The model describes time-series evolutions of magma discharge rate (Q) and pressure at the magma chamber (P). In the magma chamber, the time derivative of P (dP/dt) is proportional to the difference between magma influx to the chamber and magma outflux to the conduit (i.e., Q), and its proportionality constant is the parameter C = G/V_ch where G is the rigidity of surrounding rocks and V_ch is the chamber volume. In the conduit flow, a momentum conservation equation describes the relationship among P, Q, the magma viscosity, and the magma density. In order to take into account the effects of the viscosity and density changes, we calculated the average magma viscosity and density in the conduit under the assumptions of a stepwise increase in the viscosity and a stepwise decrease in the density during magma ascent. The positions of these stepwise changes are determined by the timescale for crystallization (t_c) and that for gas escape (t_g), and these timescales are controlled by magma properties such as crystal growth rate and magma permeability. The developed model enables us to systematically investigate how the evolutions of P and Q depend on the parameters C, t_c, and t_g.

On the basis of our model, we can obtain the relationship between P and Q in the fixed points (referred to as P_f and Q_f) in which the time derivatives of P and Q are equal to 0. The positive-feedback mechanisms by the viscosity and density changes generate a sigmoidal shape in the curve of the P_f - Q_f relationship: the slope of the curve is positive in the low-Q and high-Q regions, whereas it is negative in the intermediate region. In this case, the time-series evolutions of P and Q (i.e., trajectory) show a cyclic behavior when the fixed point in the negative slope is unstable. A notable feature of the effect of the density change on the P_f - Q_f relationship is that the value of P_f in region of the negative slope becomes much lower than the lithostatic pressure. We found that in this case, the magma discharge rate Q reaches 0 during the cyclic behavior in the time-series evolution, which may correspond to the cessation of an eruption. Because whether Q reaches 0 or not depends on the parameters C, t_c, and t_g, we can obtain a critical condition of magmatic and geological parameters for eruption cessation using our model.
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