A shallow-water model on spherical helix nodes

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1. Introduction

A variable can be expanded by Radial Basis Functions (RBF's) that depend only on distance. The expansion coefficients are computed with the collocation method, where the interpolated value at a node matches the given value. The RBF expansion provides accurate interpolation at any point in the domain. Differential operators can be constructed by differentiating the RBF interpolation equation. A differentiation operator is expressed by $BA^{-1}$the product of the matrix that is proportional to the differential of RBF and the inverse of the interpolation matrix whose elements are the RBF's. Flyer and Wright (2009) constructed a shallow water model on the sphere using RBF differential operators and demonstrated that spectral (high-order) accuracy can be obtained with their model. The nodes may be located any place on the sphere, but it is known that a quasi-uniform distribution results in higher accuracy. In this study, the RBF method is applied to the spherical helix (SH) nodes, which can be constructed easily to be quasi-uniform and compare our model with those with the minimum energy (ME) nodes used in Flyer and Wright (200) and with the icosahedral nodes used in NICAM (NI, Tomita et al. 2001).

2. Spherical Helix

Quasi-uniform nodes based on spherical helix may be generated very easily with a simple, non-iterative algorithm. A spherical helix is expressed by a simple formula between the longitude λ and the colatitude θ (the spherical helix equation, Bauer 2000)

$\lambda = m\theta \bmod 2\pi$

where m is the tilt of the helix. Here $m^2=n\pi$ and the number of the nodes is chosen to be even to distribute the nodes equally on each hemisphere. To mitigate inhomogeneity near the pole, nodes are not located on the sphere and n nodes are distributed with an equal interval in cos θ.

3. Homogeneity of nodes

The weights for the spherical surface integral represents homogeneity of nodes. The weight is proportional to the sum of the row of the inverse of the RBF interpolation matrix A^{-1}. The anomaly from the homogenous weight is computed and three node sets SH, ME, NI are compared (Fig. 1). With ME inhomogeneity is large in several areas and with NI inhomogeneity is tend to be small on the faces and edges and large on the vertices. By contrast, with SH inhomogeneity is slightly large near the poles where the curvature of the helix is large and the error is within 1% outside the polar regions.

4. Shallow water tests

The three node sets are compared with the standard tests for shallow water models (Williamson et al. 1992). The 4^{th} Runge-Kutta method is used for the time integration and the multiquadratics for RBF. In the steady state experiments (Case 2 and 3) SH rivals ME in accuracy and often with smaller errors. In the more realistic settings (Case 5 flow over an isolated mountain; Case 6 Rossby–Haurowitz Wave; Case 7 Analyzed Initial Condition), however, the differences between SH and ME are obscure. NI is less accurate in general, but excels in the conservation of vortex and divergence.

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