A Singular-Value Analysis of Perturbations Generated on a Cylindrical Shear Region around the Axial Flow

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According to the "slice method" developed by Bjerknes (1938), updrafts within a moist convection occur in infinitesimally small areas while the compensating weak downdrafts form in the remaining broader areas. Such a situation can be modeled with the axial flow surrounded by an environmental flow pointing the opposite direction, where due to the existence of a cylindrical shear region, axi-symmetric and/or asymmetric perturbations can be excited. A part of turbulence formed within moist convections may be caused by such shear existing around the strong updrafts occupying relatively small areas. In the present study, by envisioning such a phenomenon occurring in moist convections, stability of the axial flow surrounded by an environmental flow pointing opposite direction is, under the assumption of linearity, investigated first with an eigenvalue analysis. Then, the forms of optimally-excited perturbations are examined by performing a singular-value analysis.

By assuming the irrotational flow with constant density, Bernoulli's equation and the equation of continuity are adopted as the governing equations. After linearizing the equations around the basic state consisting of the axial flow and the environmental counter flow and describing the velocity with the velocity potential, two equations are solved simultaneously. Especially, applying the boundary conditions and relating the variables just inside and outside of the cylindrical shear region leads to a second order ordinary differential equation on L, i.e. the infinitesimal radial deformation of the cylindrical shear region. Then, the equation is transformed into a form of dynamical system on $X=(L, L')$, i.e. $dX/dt = iJX$, where t indicates time, $i = (-1)^{1/2}$ and J the 2×2 matrix.

The stability of the system is revealed by investigating the eigenvalue of the matrix J. Meanwhile, a resolvent matrix M which connects X with its initial value X_0 is obtained after solving the equation $dX/dt = iJX$ as an initial value problem. Then, the forward and backward singular vectors are derived as the eigenvecor of M^*M and MM^*, respectively, where M^* indicates the adjoint of M. The singular values of the system can be obtained as the square root of eigenvalues of M^*M or MM^*. Fortunately, these singular values and vectors are derived analytically. The results indicate the existence of optimally-excited perturbations which grow much faster than any corresponding eigen modes.

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