Zonal flow formation in inviscid two-dimensional Rossby wave turbulence on a rotating sphere

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Banded structure is one of the features that strongly interest us when we think about atmospheres on large planets, and therefore a lot of research has been done in order to clarify the mechanisms of the formation of zonal flows in planetary atmosphere. Barotropic two-dimensional model on a rotating sphere is one of the simplest models whose solution of the flow field has a structure with large-scale zonal flows. In unforced freely decaying systems, westward circumpolar zonal jets appear around both poles [1,2], and the scaling laws of width and the mean velocity of the polar jets of these circumpolar zonal jets have been proposed to be D ~ $\mathrm{Omega}^{-1/4}$ and U_P ~ $\mathrm{Omega}^{1/4}$, where Omega is the rotation rate of the sphere [2]. In forced systems, on the other hand, multiple zonal-band structure appears at the first stage of time integration [3, 4], and then structures with only two or three very large zonal jets are realized as an asymptotic state [5]. Many studies have been done on the zonal flows in two-dimensional turbulence on a rotating sphere, but the mechanism of the zonal flow formation in this system has not been made clear yet. In the framework of weakly nonlinear analysis, it is known that the existence of viscosity leads to the formation of zonal jets (non-acceleration theorem) [6, 7, 8], but there is no guarantee that this theory can be applied for the explanation of zonal flow formation in full-nonlinear systems. Therefore, in this study, we have examined the solutions of unforced two-dimensional turbulence on a rotating sphere with very weak viscosity to capture the inviscid limit solution in order to discuss the solution of two-dimensional Euler equation on a rotating sphere.

By performing numerical experiments on two-dimensional barotropic system with sixteen different very small viscosity coefficients, we have found the inviscid-limit solution has a structure with large-scale westward circumpolar zonal jets around both poles, which is consistent with the results of viscous cases [1,2]. Then together with the theorems for the inviscid limit solution of two dimensional Navier-Stokes equation and that of two-dimensional Euler equation [9, 10, 11], it was suggested that the solution of two-dimensional Euler equation on a rotating sphere has a structure with large-scale westward circumpolar zonal jets around both poles. This result shows viscosity is not indispensable for the formation of large-scale zonal jets in two-dimensional turbulence on a rotating sphere, and the non-acceleration theorem [6, 7, 8] in the framework of weakly-nonlinear theory can be applied for the full-nonlinear systems.

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