Planck Distribution in a Curved Spacetime

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Equilibrium distribution of a photon gas in a curved spacetime has been investigated. When a curved spacetime has a timelike Killing vector field, there exists a corresponding conserved quantity. According to relativistic thermodynamics, an equilibrium state can be achieved by a matter moving along this Killing vector. When we wish to calculate the equilibrium state of the photon gas, however, there arises one problem: the local frequency of each photon mode varies depending on the position because of the red/blue shift due to the spacetime curvature (gravity).

The aim of the present study is to calculate the equilibrium distribution, or the Planck distribution, using a generalized temperature defined from the relativistic thermodynamic. This generalized temperature is derived from the conserved quantity along the Killing vector. As a simplest example, we consider a cavity with constant acceleration; photons are defined by the mode functions in the coordinate system moving with the constant acceleration (Rindler coordinates). In this case, the conserved quantity is the one corresponds to Lorentz boost, which is called Rindler energy.

The resulting distribution has a form similar to the conventional Planck distribution with a temperature scaled by the Rindler energy, as expected. A remarkable difference is that the heat capacity depends on the position of the cavity boundary. The heat capacity increases as the boundary becomes closer to the Rindler horizon, and diverges to infinity at the horizon. This can be understood from the fact that number of states increases for the waves closer to the horizon due to the blue shift.

This simple example is for a flat spacetime, however, a general curved spacetime can be locally approximated by this calculation. Therefore, the results can be applicable for thermal radiations around neutron stars and black holes.

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