Pattern formation of injected fluid into a quasi two-dimensional cell filled with viscoelastic fluid

*Kiwamu Yoshii¹, Yutaka Sumino²

1. The University of Tokyo, 2. Tokyo University of Science

Introduction
In 1898, Henry Selby Hele-Shaw invented a quasi two-dimensional cell known as a Hele-Shaw cell to observe a flow of the fluid [1]. When a higher viscosity fluid filled in a Hele-Shaw cell, which consists of two parallel plates and has a thin gap, is replaced by a lower viscous fluid, the fractal pattern which is called viscous fingering (VF) emerges due to Saffman-Taylor instability [2]. Many materials in a daily life have properties of both viscous liquid and elastic solid, viscoelastic bodies. To grasp properties of these materials is interesting not only in theory, but also from the viewpoint of manufacturing industry. And studies of viscoelastic materials widely cover many subjects including gels, cosmetics, foods and crustal plates. In particular, when we make the experimental model of the earthquake in a laboratory, we sometimes adopt viscoelastic materials as crustal plate [3]. We focus on the crossover dynamics from liquid to solid, and vice versa, so we filled viscoelastic fluid in a Hele-Shaw cell displace another viscous fluid and observed the pattern formation.

Experiment
In this study, we used viscoelastic fluid as outer fluid which composed of worm-like micellar solution. This sample is an aqueous solution of cetyltrymethylammonium bromide (CTAB) and sodium salicylate (NaSal). This aqueous solution is known as an ideal viscoelastic fluid described by the Maxwell model with a single relaxation time [4]. To make this sample, we use an aqueous solution of CTAB mixed with NaSal at a temperature of 60 - 70 °C. Viscoelasticity of this samples was measured by rheometer (MARSIII, Thermo ScientificTM HAAKETM.), and we confirmed that the storage modulus $G'$ and loss modulus $G''$. We calculate shear modulus $G$ and relaxation time $\tau$ fitting with $G'$ and $G''$. We used a Hele-Shaw cell with the gap width of $h=0.6$ mm made of polystyrene petri dish (166058, Thermo Fisher Scientific.) as a lower plate and acrylic plate (MITSUBISHI RAYON CO.,LTD.) as a upper plate. As an inner fluid tetradecane colored with oil red was used, the Hele-Shaw cell was filled with outer fluid and stored for 24 hours. The inner fluid was injected from the center of the upper plate (Fig.1(a)). We used a syringe pump to inject the inner fluid with various injection rates $Q$. The pattern dynamics was recorded from the bottom of the cell with a digital video camera, and analyzed using Image J [5].

Result & Discussion
The interfacial motion showed temporal transition; i.e., circular growth of the interface near the inlet to the fingering pattern with characteristic size (2-3 mm) (Fig.1(b)). If this growth of the interface assume VF, the wavelength $\lambda$ depends on the interfacial tension $\gamma$, the growth speed of finger $v$, and the gap of cell $h$ [2]. However, we gained similar velocity distribution about the growth speed of finger in respective injection rate. We also obtained the characteristic wavelength of the pattern which approximately 10 times than the estimated theoretical value. If this growth of the interface results from detaching from substrates, the wavelength only depends on the gap of cell as $\lambda=4h=2.4$ mm [6]. From these results, we assume viscoelastic fluid behaves solidly in the fingering pattern, while liquidly in circular growth of the interface near the inlet.
Prospect
The plate boundary of the earth can be classified in stable slip zone, transition zone and seismogenic zone[7]. The transition zone where slow earthquakes occur has properties of both brittle and ductile[8]. We try to create the experimental system of crustal plate using another viscoelastic fluid, which has different rheological behavior.

Reference

Keywords: rheology, viscoelasticity

(a) Setup of Hele-Shaw cell and experiment system. (b) Snapshot of fingering pattern growth where $Q = 9.0$ ml/min.