Contact theory based power-law relation representing the pressure dependence of elastic wave velocity

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The pressure dependence of elastic wave velocity through fractured media has been of great interest for many decades. Abundant experimental studies have been conducted to observe the velocity variation (e.g. Birch, 1960) and theoretical explanations have provided the elastic wave velocity behavior under increasing pressure (e.g. Walsh, 1965). However, the pressure dependence of wave velocity was not well established until Eberhart-Phillips et al. (1989) introduced an empirical relationship of,

$V = V^* + KP - B\,exp(-DP)$, (1)

where V is velocity and V^*, K, B and D are constants for a given rock type. Since the relationship was derived using velocity measurements of Han (1986) conducted under the pressures less than 40 MPa, the reliability of relationship (1) at higher pressure is questionable and this was in fact discussed by later studies (Jones, 1995; Khaskar et al., 1999). Jones (1995) showed that this relationship is unable to predict the quality factor of P-wave and S-wave at high effective stress, and Khaskar et al. (1999) showed that it is unable to represent the negative K at elevated pressures. Moreover, this can only be applied to sedimentary rocks. Therefore, we need other relationships, which can represent the pressure dependence of elastic velocity at both low and high pressure independent of rock types.

We developed a power-law relation combining Nagumo (1963)'s contact state relationships for different contact shapes and a one-dimensional longitudinal elastic wave velocity equation based on the contact theory of Hertz (1881). The obtained power-law is in the form of,

$V^2 = V_0^{\,2} + A' P^{\mu}$, (2)

where P, V, V_0, μ and A' are the effective pressure, wave velocity at given and zero effective pressures, the multiple contact state and a pressure dependent variable, respectively. The relationship is similar to Kobayashi and Furuzumi (1977)'s empirical relationship with different meanings to the parameters, especially A' which was shown as a constant.

Experimental and theoretical studies have shown that the pressure dependence of the elastic wave velocity highly depends on the fracture openings in a rock. Therefore, the multiple contact state in the present power-law relation is an important parameter as it enables to represent the velocity variation with respect to fracture openings. The two end values of μ represent the fracture open (μ =2/3) and close (μ =1/2) conditions while the values in between show partly open state of fractures. Any μ value greater than 2/3 indicates open fractures which appear in low-pressure regions until the fractures gradually close and increase velocity. Once the fractures are closed the velocity increases intrinsically and this is represented by μ values less than 1/2. In rocks with high porosity, after reaching a steady velocity increment at high pressure, velocity drops result by pore collapsing (e.g. Fortin et al., 2007). This velocity drop is represented by the change of contact state from a μ value less than 1/2 to a value greater than 2/3 which indicates fracture opening at pore collapse. Even though relationship (1) represents the velocity variation at low pressures accurately, since it is unable to show such variations at high pressure, relationship (2) becomes a more reliable representation of the pressure dependence of elastic wave velocity. While μ gives the multiple contact state, the physical properties of the contacts (size and shape) and the elastic properties (Young's modulus and Poisson's ratio) of the rock are given by A' which explains the pressure dependence at high pressure. Therefore, this relationship provides advanced information on the rock properties. However, this also has limitations since it can only be used for rocks with low fracture densities ($\Phi < 0.5$) and isotropic fracture distribution. Thus, as an alternative to the empirical relationship (1), we introduce the derived power-law (2) which can represent the velocity at both low and high pressures for any given rock type.

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