On the possible bifurcation mechanisms and associated dynamical properties of Dansgaard-Oeschger oscillations

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Dansgaard-Oeschger (DO) oscillations are millennial-scale abrupt climate changes in the last glacial period (Dansgaard et al. 1993; Johnsen et al. 1992). In particular, they are frequent in the mid-glacial and less frequent at the beginning of the glacial and in the Last Glacial Maximum (McManus et al. 1999; Kawamura et al. 2017). Since its discovery, DO oscillations have been intensively studied, but its detailed mechanisms are still a subject of debate. Our recent modeling study (Abe-Ouchi et al. in prep) with MIROC4m Atmosphere-Ocean general circulation model (Hasumi and Emori 2004) suggests that DO oscillations are spontaneous oscillations which arise from global-scale interactions between the atmosphere and ocean (see also studies in [12] and Brown and Galbraith, 2016). Our simulated oscillations are likely to appear in a certain region of background climate conditions directly related to factors such as ice sheets, orbital elements, and greenhouse gases (Abe-Ouchi et al. in prep; Sam Sherriff-Tadano et al. 2017). This result is consistent with the observations that DO oscillations were most frequent in the mid-glacial.

The bifurcation theory is a field of mathematics which studies qualitative changes of dynamical systems depending on the changes in parameters. In general, we can consider numerous types of dynamical systems, but their bifurcation phenomena are classified into only several types (Strogatz, Nonlinear Dynamics And Chaos, 2000). Identifying bifurcation mechanisms may help us to understand the emergence mechanism of the oscillation and its associated properties like the period or the amplitude. For example, when oscillations emerge/vanish via a Homoclinic bifurcation, the period of oscillations diverges to infinity as the parameter approaches the bifurcation point. On the other hand, when oscillations emerge via a Hopf bifurcation, such divergence of the period does not occur. On the basis of such scaling properties of the period, previous studies [7, 9] suggested onset mechanisms for the oscillations.

In this study, we survey bifurcation mechanisms reported in previous models of DO oscillations [1-12]. We also performed a bifurcation analysis of an extended version of the Winton (1993) 3-box halocline oscillator [2], which has a parameter dependence similar to that of MIROC AOCGM. It is found that, in many models, DO-like oscillations emerge via a Hopf-type bifurcation [1,2,4,5,6,7,10,11] and/or via a Homoclinic-type bifurcation [1,2,3,5,6,7,8,9,11]. The Welander (1982) model [1] and the extended Winton model may generate oscillations via both a Hopf-type bifurcation and a Homoclinic-type bifurcation depending on their parameters, but these bifurcations become identical at a parameter limit. This implies that the oscillation period may increase rapidly even near the Hopf bifurcation points (though it does not diverge). Therefore, identifying onset mechanisms of oscillations on the basis of scaling properties of the period may have a subtle problem. The extended Winton's model has a phase space structure like Homoclinic orbit even near the Hopf bifurcation point, and this phase space structure induces a long oscillation period. In MIROC AOGCM, the onset of oscillation may be related to either a Hopf-type bifurcation or a Homoclinic-type bifurcation, or to both of them. The determination of the bifurcation type in MIROC AOGCM is still difficult because of limited computational time and inherent natural variability. Nevertheless, we observe that noisy 'limit cycles' near bifurcation points show characteristics similar to Homoclinic orbit and have relatively long periods.

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