Improvement for Finite-Difference Formula of Non-Linear Longwave Equations to Express Tsunami Decay

*Masaaki MINAMI[1]

1. Meteorological Research Institute

In the current, we can not correctly express the decay process of the tsunami by numerical calculation. In the meantime, social demands for accurately expressing the tsunami decay are strengthening, for judging the timing of cancellation of the tsunami warning and information on the outlook of the tsunami. Therefore, at the Meteorological Research Institute, we are conducting research to improve the accuracy of the subsequent wave and decay process of the far-field tsunami and modeling of the decay process using observation waveforms (Hayashi et al., 2010, JSCE papers B2 (coastal engineering) etc.). We are conducting large-scale and highly accurate tsunami calculations using supercomputer using nonlinear longwave equations, which is a conventional tsunami numerical calculation method. (Minami et al., 2018, Japan Seismological Society Autumn Meeting etc.) Nevertheless, the decay process of the tsunami was not correctly expressed like the results of various calculations so far.

In this paper, we focused on bottom friction of tsunamis, which is an important factor in decay, and improved the differential formula of nonlinear longwave equations. First, in the friction term in the equation, the position difference of the total water depth (D) for calculating the friction term has been calculated as the arithmetic mean of the two adjacent meshes (central difference). If you use arithmetic averaging, there is a possibility that non-negligible errors may occur compared to when solving continuously. This is because D changes in the friction term on the order of -1/3. Therefore, the positional difference was improved to the one using the average value theorem (fig1). Furthermore, with regard to the time difference, assuming that *dt* is *dt/n* within one calculation step *dt* seconds and assuming that the calculation is repeated *n* times, it is possible to take the limit at $n \to \infty$, and the difference expression can be transformed as shown in fig 2.

Using these improved difference formula and appropriate coefficients, we calculated in the case of the 2010 Chile earthquake tsunami (Mw 8.8), and as a result its deay is larger than the conventional formula. In the example of fig3, the total energy amount of the later part, which became larger than the observation amount, decreased by about 4% and approached the observation amount. As a calculation condition of the tsunami, all areas including the entire Pacific Ocean (E100° to W90°) are assumed to be 30 seconds mesh (GEBCO 30 sec-grid), and in the area surrounding Hokkaido, Honshu, Shikoku and Kyushu, nesting is performed with a 10 second mesh, and we calculated for a long time (72 hours) in order to see the decay process. For the calculation we use JAGURS (Baba et al., 2015) , considering the elastic loading of the crust and the density effect of seawater, and using the result of the fault parameter (Fujii & Satake, 2013).

By using this formula, the decay of the tsunami becomes larger than the conventional formula by about 5%, and it is 10% or more in large case when the total depth D is small (generally less than 1 m) such as run-up and exposure. Also, by improving the time difference, it is possible to take longer time steps and it is possible to shorten the calculation time.

Although the decay process of the tsunami was not correctly expressed by the previous numerical calculation, it became possible to express the decay process more correctly by this method. In the future, we will continue to study this method and apply this method to various cases to improve the accuracy of the tsunami decay process.

Keywords: Far-field tsunami, Numerical simulation, Supercomputer

$$gn^2\left(\frac{D^k_{i,j}+D^k_{i+1,j}}{2}\right)^{-7/3}\frac{M^{k+1/2}_{i+1/2,j}+M^{k-1/2}_{i+1/2,j}}{2}\sqrt{(M^{k-1/2}_{i+1/2,j})^2+(N^{k-1/2}_{i+1/2,j})^2}$$

$$\downarrow$$

$$gn^2\frac{3}{2}\frac{(D^k_{i+1,j})^{\frac{2}{3}}-(D^k_{i,j})^{\frac{2}{3}}}{D^k_{i+1,j}-D^k_{i,j}}\left(\frac{D^k_{i,j}+D^k_{i+1,j}}{2}\right)^{-2}\frac{M^{k+1/2}_{i+1/2,j}+M^{k-1/2}_{i+1/2,j}}{2}\sqrt{(M^{k-1/2}_{i+1/2,j})^2+(N^{k-1/2}_{i+1/2,j})^2}$$

fig1. Bottom Friction Term Using Mean Value Theorem

Conventional difference formula is

$$\text{M\_new(i,j)} = (\text{M}'\text{(i,j)}-\frac{1}{2}\text{fric}*\text{M\_old(i,j)}) / (1+\frac{1}{2}\text{fric})$$

M′ (i,j)=M_old(i,j)−(gravity term) – (advection term)

fric= dt・g・n^2・D^-7/3・$\bar{M}$

solving n times

$$\text{M}_{1/n}\text{(i,j)} = \text{M}'\text{(i,j)}-\frac{1}{n}*\text{fric}*\text{M\_old(i,j)}$$

$$\text{M}_{2/n}\text{(i,j)} = \text{M}_{1/n}\text{(i,j)}-\frac{1}{n}*\text{fric}*\text{M}_{1/n}\text{(i,j)}$$

$$\vdots$$

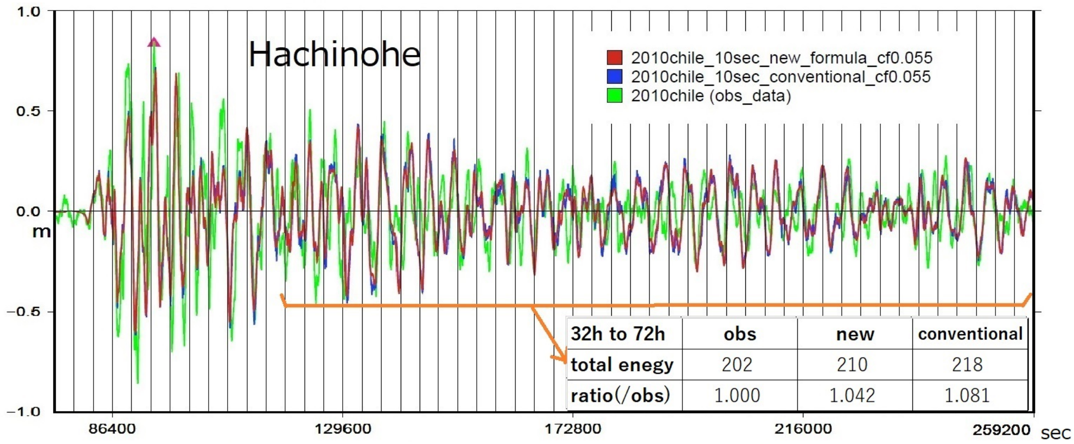
$$\text{M}_{n/n}\text{(i,j)} = \text{M}_{n-1/n}\text{(i,j)}-\frac{1}{n}*\text{fric}*\text{M}_{n-1/n}\text{(i,j)}$$

$$\text{M\_new(i,j)} = (\text{M}'\text{(i,j)}-\frac{1}{n}*\text{fric}*\text{M\_old(i,j)})\cdot(1-\frac{\text{fric}}{n})^{n-1}$$

n→∞, (1-a/n)^n=exp(-a)

$$\text{M\_new(i,j)} = \text{M}'\text{(i,j)}\cdot\exp(-\text{fric})$$

fig2. Bottom Friction Term Using Limit of a function



32h to 72h	obs	new	conventional
total enegy	202	210	218
ratio(/obs)	1.000	1.042	1.081

fig3. Time Series of Comparisons between New and Conventional Formula