# Axisymmetric disturbance on an axisymmetric shallow water vortex 

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The air in an eye of a typhoon is often considered to be stratospheric origin. Indeed, the region of high equivalent potential temperature invading eyes from stratosphere can be seen in most of tropical cyclones. Therefore, we can recognize tropical cyclones are vortices where the air with low potential temperature revolves around the warm air sucked from stratosphere. Also, elliptical or polygonal deformation of an eye of a tropical cyclone can be attributed to the oscillation of the interface between such two kinds of fluids. Thus, in this study, a tropical cyclone is modeled with two layer fluids where the upper fluid touches the ground surface at the core region of the vortex, and the motion on the interface is investigated with the shallow water equations described in polar coordinate by assuming the upper fluid at rest.
An axisymmetric swirling fluid whose center is completely dried up is adopted as the basic state, where the tangential speed is assumed to be inversely proportional to the $\alpha$-th power of radius. Perturbations are again set to be axisymmetric. Under these settings, the governing equations are linearized, and an eigenvalue problem is formulated after hypothesizing sinusoidal time variation with the angular frequency $\omega$. The eigenvalues are, at first, obtained numerically by discretizing the variables with a standard staggered grid. Then, in the case of no ambient rotation, the problem is solved analytically, by applying a series expansion to the governing equations.
Results reveal the existence of modes like gravity waves. Since the amplitudes of the modes are non-zero at the inner boundary corresponding to the outer edge of the dry core, they would be responsible for the periodic expansion and contraction of an eye in an actual tropical cyclone. Moreover, quasi-steady modes with no nodes in the radial direction emerge in the case of no ambient rotation. Although numerical and analytic solutions roughly coincide with each other, a definite discrepancy is found near the dry core of the basic vortex: radial cross sections reveal that the numerical solutions tend to swing once more against the analytic solutions around there.

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