Two-dimensional or shallow water MHD waves on a rotating sphere with an imposed azimuthal magnetic field

*Ryosuke Nakashima[1], Shigeo Yoshida[2]

1. Department of Earth and Planetary Sciences, Graduate School of Science, Kyushu University, 2. Department of Earth and Planetary Sciences, Faculty of Science, Kyushu University

Magnetohydrodynamic (MHD) waves in a thin layer on a rotating sphere with an imposed toroidal magnetic field are investigated. The system is often considered as a model of the stably stratified outermost Earth's outer core or the tachocline of the Sun. The stratification of the outermost core is suggested on the basis of seismological evidence (e.g. Helffrich and Kaneshima, 2010; Kaneshima, 2018) and interpretations of the geomagnetic variations with MHD waves (e.g. Braginsky, 1993; Buffett, 2014; Chulliat et al., 2015). In order to provide constraints on the obscure stratified layer by comparing with wavy variations in the geomagnetic field, we studied the linear waves of the two-dimensional MHD and the MHD shallow water system over a rotating sphere.

We adopt an azimuthal equatorially antisymmetric field ($B_\Phi(\theta) = B_0 \sin\theta\cos\theta$, where θ is colatitude, Φ is azimuth) as a background magnetic field. On the other hand, an equatorially symmetric field ($B_\Phi(\theta) = B_0 \sin\theta$) was assumed in Márquez-Artavia et al.(2017), whose results we replicated and reported in JpGU 2018.

Compared with previous results, the dispersion diagrams obtained with the toroidal equatorially antisymmetric field show that some fast magnetic Rossby branches remain, while slow magnetic Rossby waves disappear. Besides, a continuous spectrum is found in the range where an azimuthal phase velocity is coincident with a local Alfvén velocity divided by $\sin\theta$. Similar continuous spectra are also seen in various physical situations, including inviscid shear flow (e.g. Case, 1960; Balmforth and Morrison, 1995; Iga, 2013) and plasma oscillations (e.g. Van Kampen, 1955; Case, 1959; Barston, 1964; Sedláček, 1971). The continuous spectra are accompanied by a singular eigenfunction, which is physically meaningful only when they are integrated over the continuous spectra. Its integrated solutions generally decay with time, which is referred to as phase mixing. Unlike exponentially damped discrete modes, this decaying is proportional to negative powers of time.

In the case of the shallow water system with the antisymmetric field, discrete eigenvalues buried in the continuous spectrum is found, which include unstable modes. For the Earth-like parameters, polar trapped modes with decadal period and equatorial trapped Rossby waves with a few years period are found when the stratification is weak.

Keywords: MHD waves, MHD shallow water, toroidal magnetic field, continuous spectra, a stably stratified layer at the top of the Earth's outer core

