Solution Orbit for k and epsilon Model for Turbulent Flow in Porous Media

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Turbulent flow within porous media such as fault rocks can affect dynamic earthquake rupture processes and has attracted interests of many researchers. Among models treating the turbulent flow, the k-epsilon model has widely been employed, where k is the turbulent kinetic energy and epsilon is the dissipation rate of k. However, analytical treatment associated with the effect of the initial values of k and epsilon on the final state has not been performed.

We assume a one-dimensional homogeneous porous medium and isotropic turbulence. We also assume that the density of the fluid, rho, and the porosity, phi, are constant, and the ensemble averages of phi k and phi epsilon will be written as k and epsilon below, respectively. With these assumptions, it should be emphasized that exactly the same straight line given by the equation epsilon=c_k u_D k /\sqrt{*K*} is a nullcline common to both variables on the k-epsilon phase space, where c_k is a constant number, u_D is averaged Darcy velocity, and *K* is permeability. We can regard this line as a line attractor or repeller as observed in other systems such as Suzuki (2017). Moreover, the straight line epsilon=0 (k axis) is also a nullcline for epsilon.

Actually, the analytical form of solution orbits is given by epsilon=epsilon$_0$ (k/k_0)^C_2, where C_2 is a constant, 1.9. Using this, we show below that the common nullcline epsilon=c_k phi u_D k/\sqrt{*K*} is a line attractor for this solution orbit. First, we define Region I as the region 0< epsilon < c_k u_D k /\sqrt{*K*} on the phase space, and Region II as the region epsilon > c_k u_D k/\sqrt{K}>0. We also define the point (k_f, epsilon$_f$) as the point where the solution orbit passing the point (k_0, epsilon$_0$) crosses the nullcline. With these definitions, we have relations k_0f and epsilon$_0$f if (k_0, epsilon$_0$) is in the Region I. This occurs because epsilon is proportional to k on the nullcline, while epsilon is proportional to k^C_2 on the solution orbit and C_2 >1. We can also conclude k_0>k_f and epsilon$_0$>epsilon$_f$ if (k_0, epsilon$_0$) is in the Region II. Second, we should emphasize that k and epsilon increase (decrease) with increasing time in the Region I (II), since \partial k/\partial t and \partial epsilon/\partial t are positive (negative). Therefore, if (k_0, epsilon$_0$) is in the Region I (II), the solution moves to the upper right (lower left), and is absorbed into the nullcline with the limit t \to \infty. We can conclude that the nullcline is a line attractor, not a repeller. The steady stable solution is given by (k, epsilon)=(k_f, epsilon$_f$).

Note that k and epsilon vanish with the limit t \to \infty for usual isotropic turbulent flow, even though the solution orbit for such flow is the same as one obtained here. Actually, the usual turbulent flow is described by the limit *K* \to \infty in the present model. The nullcline is k axis in such a case, and the Region I vanishes. Therefore, all the solutions are absorbed into the origin. The finite *K* enables the turbulence to be survive with t \to \infty for the homogeneous state.

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