The time-fractional diffusion equation as the governing equation of the concentraiton of Cs-137 in the environment

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After almost 10 years of the Fukushima accident, measurement data have been accumulated in various environment. We propose a new model on how the radioactive cesium decreases in the long time such as a decade, other than the box model. The box model, for Cs-137 in the lake water for example, assumes instantaneous, homogeneous mixing of solute and water, two or three boxes are connected with each other to reproduce the data of the Cs-137 concentration (IAEA-TECDOC-1143, 2000). Transfer coefficients, k, are used in the box model, that if the value of k's are constants, the exponential function exp(-kt) is intrinsic for the behavior of the concentration. In case of several boxes, the concentration should follow the sum of these exponential functions as A exp(-k1 t) + B exp(- k2 t) + ... There have been critics among several researchers that this model implies a "fatalist" attitude; it means that from the initial stage, each radionuclide is designated for its process to follow. In particular, we would meet the situation that the second exponential function appears after we have conducted a couple of years of measurement. This study is intended to improve such situations. We thus propose a new model with a time-fractional diffusion equation. Because the actual data decrease in time in such a manner that they decrease very rapidly with the exponential function at first; then, several years later, they decrease more slowly as the power function. This behavior is very similar characteristics of Mittag-Leffer function, which is the fundamental solution of the time-fractional diffusion equation (Cauchy problem). The significance of fractional time-derivative is that it can cope with the "memory" effect that every cesium particle has, the flux of cesium at a specific time at a specific location depends on the previous concentration of cesium as a power law of the time. We use the Mittag-Leffler function to reproduce actual measured concentration of Cs-137. We note that Bulgakov et al.(2002)'s study yields an semi-analytical solution of the lake concentration of Cs-137 in Bryansk, Russia as $\exp(-\lambda t)/\sqrt{t}$, which is a special case of the Mittag-Leffler function when its parameter alpha=1/2. It seems that the fractional derivatives have been used for other problems such as the geothermal energy productions (A. Suzuki et al.) and solute transport in porous media (B. Berkowitz et al.).

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