A semi-Lagrangian advection model on the sphere using radial basis functions

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1. Introduction

Radial Basis Functions (RBF) depend only on the distance from the node. Flyer and Wright (2007)[1] developed an advection model using RBF and showed that the spectral accuracy can be achieved in advection tests. The minimum energy nodes employed in the previous studies require iterative solution which may not converge to the global minimum. Therefore, the author instead used spherical helix nodes, which can be obtained deterministically and showed advantages in uniformity (JpGu 2018 MGI28-04). RBF allows accurate interpolation from data on scattered nodes. In this study, a semi-Lagrangian advection model with RBF interpolation has been developed to compare against the Eulerian advection model.

2. Model

Over the sphere 4096 spherical helix nodes[2] are used. The Eulerian model uses the derivative operator for solid body rotation[1]. Gaussian RBF with the shape parameter of 10 is used. The time step is 30 minutes. The semi-Lagrangian model uses upstream search in the Gaussian coordinates[3] and RBF interpolation. Multiquadric and Gaussian RBF's are used with shape parameter of approximately 8 calculated from the node interval. The time step is 90 minutes, three times of that in the Eulerian model.

A cosine bell is advected with a slant solid body rotation to pass the poles[4] and the difference of the distribution after one revolution from the initial distribution is regarded as error.

3. Results

The semi-Lagrangian model is somewhat more accurate than the Eulerian model. The L_2 norm is halved with Gaussian RBF in particular. In addition small but omnipresent noise over the globe in the Eulerian model is eliminated and error is localized around the cosine bell in the semi-Lagrangian model.

4. Summary and discussions

RBF enables accurate interpolation on scattered quasi-uniform nodes in the semi-Lagrangian advection. Equivalent or better accuracy is achieved in the semi-Lagrangian model while using a longer time step than that in the Eulerian model. The inverse of the interpolation matrix does not have to be computed every time step because the node distribution is time independent.

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References

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