Local node refinement using the Schmidt transform for RBF advection models

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Large-scale geophysical fluid motions are dominated by the advection process. Derivatives with rapid change using Eulerian advection require high resolution node. In this study, local node refinement was assessed in the test case in which two vortices roll-up on the sphere.

We approximated a spatial derivative operator with Radial Basis Functions (Flyer and Wright 2007)[1]. Radial Basis Functions (RBF) has the following useful characteristics. 1) RBF's are not linked to any coordinate systems because RBF's depend only on the distance between from node. For example, RBF's are free from the singularities of the derivative operators at the poles in spherical coordinates. 2) An RBF method is easy to perform calculation and does not increase complexity at higher dimensions. 3) The node placement is flexible to allow the use of more uniform nodes on the sphere.

Spectral accuracy can be achieved in advection on quasi uniform nodes. However, increase in the number of nodes make the condition number of interpolating matrix larger, therefore accurate calculation with high resolution nodes is not trivial. Thus, local refinement is attempted in this study.

Flyer and Lehto (2010)[2] constructed locally refined nodes near the vortex center from minimum energy nodes with electronic repulsion and showed that local refinement improves accuracy. However, the node generation with electronic repulsion that uses iterative numerical optimization may suffer from a convergence problem. Therefore, Schmidt transform that refines nodes deterministically is adopted. the test of vortex roll-up was conducted to investigate the relationship between the magnification parameter and accuracy, and improvement with the local node refinement. The shape parameter is varied by according to the minimum distance between nodes. Varying the shape parameter and improvement of the condition number with node refinement. However, excessive node clustering makes the maximum shape parameter too large. Consequently, excessive node clustering deteriorate approximation of derivative matrix. In the test case with local nodes refinement,

accuracy is improved by a factor of 7 approximately in normalized error l1 and l2.

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Reference

[1] Flyer, N., G.B. Wright, J. Comput. Phys. 226 (2007) 1059–1084.

[2] Flyer, N. and E. Lehto, J. Comput. Phys 229 (2010) 1954–1969.

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