Solving slow-velocity flow fast: An explicit method for mantle convection

*Yoshifumi Kawada[1]

1. International Research Institute of Disaster Science, Tohoku University

Fluid flow slower than the sound velocity is pervasive in the Earth's interior. Typical examples are mantle convection, outer core convection, crustal deformation, and hydrothermal circulation. Obtaining the flow field of such systems requires numerical calculations because they usually contain complex physics including chemical reactions. Among the above-described systems, this presentation focuses on mantle convection as a representative of a slow flow system and considers an effective method for solving the flow field.

The flow of mantle is hard to solve because of the following three characteristics (e.g., Takeyama et al., 2017): Flow is very slow compared with the sound velocity; fluid is highly viscous; the viscosity varies at many orders of magnitudes. In usual, the sound velocity is approximated to be infinite (or the time derivative term of the continuity equation is ignored) to remove the time step restriction caused by the fast propagation of sound. In addition to the incompressible approximation described above, the time derivative term of the momentum equation is also ignored to remove the time step restriction caused by fast viscous diffusion compared with thermal diffusion. This second approximation is unique for mantle convection. In spite that these two approximations remove the time step restriction, the continuity and momentum equations without the time derivatives must be solved iteratively at each time step of the energy equation, the only time-dependent equation.

One of the most well-known methods for obtaining the flow field of the mantle is a class of the SIMPLE method (Patankar, 1980), which solves for the flow field by calculating pressure and velocity fields alternately by sequential iterations. The method of Kameyama et al. (2005) solves for the flow field as a steady-state of the continuity and momentum equations using the pseudo-compressibility method combined with the multi-grid method. Obtaining the flow field with a variable viscosity requires care in iterative methods. Recently, Takeyama et al. (2017) adopted a completely different approach for approximating the time derivative terms of the continuity and momentum equations. The direction of approximation is opposite to that used in the existing methods: The sound velocity is slowed down near to the flow velocity of the mantle, and; the viscous diffusion is also decreased near to the thermal diffusion. With the help of these approximations, the equations can be integrated explicitly with a practically large time step.

This presentation reconsiders Takeyama et al.'s (2017) explicit method with a series of two-dimensional finite difference calculations. The equations to be solved are based on those with the Boussinesq approximation, although Takeyama et al. (2017) solved the equations for a compressible system that are not similar to those for mantle convection. The system of equations in the non-dimensionalized form has the following three parameters: the Prandtl number (Pr), the non-dimensionalized sound speed (1/M), as well as the Rayleigh number (Ra). The present explicit method is able to reproduce the steady-state benchmark problems in Blankenbach et al. (1989) at a certain parameter range of Pr and M. However, to match the time-dependent behaviors (e.g., oscillative solutions), Pr should be many orders greater than 1 but M is not sensitive to the result. The time step becomes small when Pr is large because it is proportional to the inverse of Pr. What is worse, the time step is also proportional to the square of mesh spacing in such cases. As a result, when time-dependent problems are considered with small mesh spacing, the present method becomes slow (but it may be still faster than the SIMPLE-like methods).

To overcome the small time step restriction of the explicit method with large Pr, a little more strategy is required. A class of the explicit Runge-Kutta method (e.g., Hairer et al., 1991) is known to increase the time step of diffusion equations, which may be suitable for the present situation. This method has been successfully applied to problems in magneto-hydrodynamics with highly anisotropic thermal diffusion (e.g., Vaidya et al., 2017). Here, this method is added to the explicit solver for the continuity and momentum equations of mantle convection. A series of calculations shows that this method works effectively to speed up the calculation even if Pr is very large. It is examined how fast this method can accelerate the explicit solver.

Keywords: explicit method, mantle convection, Runge-Kutta method, incompressible fluid, numerical method

