Shallow water MHD waves trapped in the polar regions on a rotating sphere with an imposed azimuthal magnetic field

*Ryosuke Nakashima[1], Shigeo Yoshida[2]

1. Faculty of Sciences, Kyushu University, 2. Department of Earth and Planetary Sciences, Faculty of Science, Kyushu University

Magnetohydrodynamic (MHD) shallow water linear waves are examined on a rotating sphere with a background toroidal magnetic field expressed as $B_{0\Phi}=B_0\sin\theta$, where B_0 is constant, θ is the colatitude and Φ is the azimuth. The MHD shallow water equations are often used in studying the dynamics of the solar tachocline (e.g. Gilman & Dikpati, 2002[1]; Márquez-Artavia et al., 2017[2]) and sometimes the outermost Earth's core (Márquez-Artavia et al., 2017[2]; Nakashima, Ph.D. thesis, 2020[3]) and exoplanetary atmosphere (e.g. Heng & Workman, 2014[4]). In this poster, we especially focus on the propagation mechanisms and the force balances of polar trapped waves and unstable modes (Márquez-Artavia et al., 2017[2]; Nakashima, Ph.D. thesis, 2020[3]).

Comprehensive searches for eigenmodes yield two polar trapped modes when the main magnetic field is weak (the Lehnert number $\alpha=V_A/2\Omega R^2<0.5$, where V_A is the Alfvén wave velocity, Ω is the rotation rate and R is the sphere radius). One is the slow magnetic Rossby waves, which propagate eastward for zonal wave number m≧2 (Márquez-Artavia et al., 2017[2]). As the Lamb's parameter $\varepsilon=4\Omega^2R^2/gh\to 0$ (where g is the gravity acceleration and h is the equivalent depth), these branches asymptotically approach the eigenvalues of two-dimensional slow magnetic Rossby waves. Another is newly discovered westward polar trapped modes (Nakashima, Ph.D. thesis, 2020[3]).

In the case when $\alpha>0.5$ (the background field is strong), these novel westward modes merge with the westward-propagating fast magnetic Rossby waves. In addition, only when m=1, polar trapped unstable modes appear due to the interaction between these fast magnetic Rossby waves and westward-propagating slow magnetic Rossby waves. These growth modes are believed to be the polar kink (Tayler) instability (Márquez-Artavia et al., 2017[2]).

In order to easily understand the propagation mechanisms and the force balances of polar trapped modes, we investigate a cylindrical model around a pole with an artificial boundary condition. This model provides the approximate dispersion relations and eigenfunctions of polar trapped modes, and indicates that stable polar trapped modes are governed by magnetostrophic balance and that the metric magnetic tension force causes the difference between the slow magnetic Rossby waves and the novel westward modes. For m=1 and $\alpha>0.5$, the balance between Coriolis and Lorentz forces is disrupted and the part of magnetic tension with which Coriolis force can not compete induces kink instability.

[Reference]

[1] Gilman, P. A., Dikpati, M. (2002) *Astrophys. J.*, **576**, 1031. doi: 10.1086/341799
[2] Márquez-Artavia, X., Jones, C. A., Tobias, S. M. (2017) *Geophys. Astrophys. Fluid Dyn.*, **111**, 282. doi: 10.1080/03091929.2017.1301937
[3] Nakashima, R. (2020) Ph.D. thesis, Kyushu University. http://dyna.geo.kyushu-u.ac.jp/HomePage/nakashima/pdf/doctoral_thesis.pdf
[4] Heng, K., Workman, J. (2014) *Astrophys. J. Sup.*, **213**, 27. doi: 10.1088/0067-0049/213/2/27

Keywords: MHD shallow water, toroidal magnetic field, polar trapped waves, slow magnetic Rossby waves, polar kink instability

