Comparison of Fluid Resistance and Experience for Calculation of Resistance Coefficient according to Object Shape

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At first, we measured air resistance with variation of shape by dropping physical objects from a height of 15.75 meters, the fifth floor of our school building. As Experiment 1, we prepared four kinds of wooden objects such as cube, quadrangular pyramid, cone and sphere with similar volume (Table 1). The result of the experiment includes errors caused by deficiencies of shape varieties and data, being inhomogeneous of density of wood, inaccuracy of shape, and by rotation of falling object around its horizontal axis. For the purpose of improving these problems we used the same settings as Experiment 2. We prepared six kinds of objects adding column and rectangular parallelepiped made by a 3D printer with a filament of PLA resin (Table 1). The problem of rotation of the falling object was solved by passing a nylon thread through the center of each object. According to the above two experiments, the resistance depends on the front shape of the object. A globe has the smallest value of resistance, followed by pyramidal or conic shape and a flat shape has the largest value. The order corresponds to the tendencies in textbooks for senior high school students and college students.

Next, we focused on the cones in the above six kinds of shapes. Cones are one of the most useful shapes in daily life. We can easily reproduce several patterns of apex angles. Therefore, we attempted to estimate the values of the fluid resistance coefficient in conjunction with the apex angles using two methods. One method is hitting the object with fluid and the other is calculating the coefficient from the terminal velocity of the falling object. We called these Experiment 3 and 4, respectively.

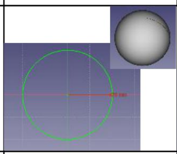
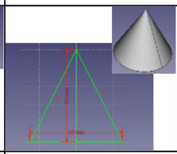
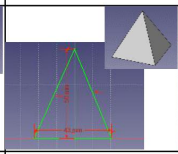
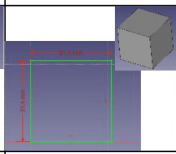
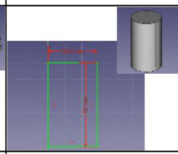
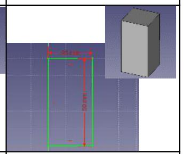
In Experiment 3, eight types of objects (Table 2) with different central angles were made with the 3D printer. The cones were suspended by spring scales, and we poured water fallen freely on them from a position a certain distance away from the objects. When water is allowed to fall freely on the top of an object, the pressure applied to the object is measured in gram weight units, and the pressure can be calculated. The resistance coefficient was calculated from the experimental results, and a graph corresponding to the change in the central angle was created (Fig 1). Fig 1 shows a curve in which the resistance coefficient increases exponentially with the increase in the central angle. Also, the resistance coefficient obtained in the experiment was displayed using function display software (FunctionView Ver 6.02), and the approximate function was predicted to obtain the equation to calculate resistance coefficient. Exponential function expressions are cumbersome and the accuracy of the approximate expression is not known. To simplify the calculation and use the standard deviation to estimate the expression more accurately, a first-order approximation using logarithms is used. An approximation using logarithms to make calculations is easier and to use the standard deviation estimates a more accurate expression. Further, the value of the resistance coefficient obtained in Experiment 3 was lower than the value described in the above reference book or the like. As the reason, we thought to be that the area that the dropped water hits is smaller than the projected area of the object, and that there is a diffrence in the force between "the object moving in the fluid" and "the fluid hitting the stopped object".

In Experiment 4, we prepared cones with no bottoms (hollow cones) with different apex angles made of copy paper (PPC paper) of uniform thickness. And we experimented in the same place in the author's house for the whole experiment (no wind, indoors). After dropping the object from a height of 2.5 meters above the floor, the speed reached terminal velocity at 2.0 meters above the floor. We measured the time taken from 2.0 meters above the floor to the landing. The terminal velocity is calculated from the measured time, and the resistance coefficient is obtained. In Experiment 4, as in Experiment 3, we will create a graph which shows relationship between central angle and resistance coefficient, and predict approximated equation using the function software.

Keywords: Fluid dynamics, Air resistance, Resistance coefficient, Cone

表 1　実験 1・2 で用いた各物体の概形及び体積（実験 1(Experiment 1)：1～4　実験 2(Experiment 2)：1～6)

Table 1　Rough shapes, its vertical sections and volumes of each object used in the Experiments 1 and 2.

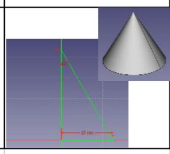
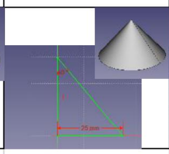
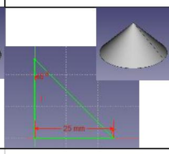
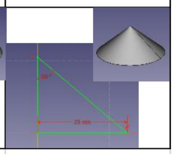
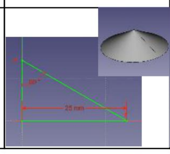
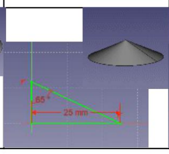
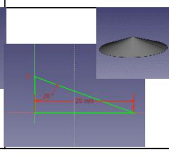
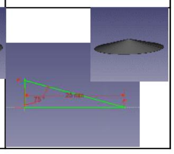
番号(No.)	1	2	3	4	5	6
形状名 (shape name)	球 (sphere)	円錐 (cone)	四角錐 (quadrangular pyramid)	立方体 (cube)	円柱 (column)	直方体 (※1)
概形※2 (rough shape)						
体積 (volume)	(4/3)×3(円周率) ×2^3 ＝32.00〔㎤〕	2.5^2×3(円周率) ×５×1/3 =31.25〔㎤〕	4.3^2×5×1/3 =30.82〔㎤〕	3.14^3 =30.96〔㎤〕	1.44^2×3(円周率)×5 =31.10〔㎤〕	2.5^2×5 =31.25〔㎤〕

※1　rectangular parallelepiped

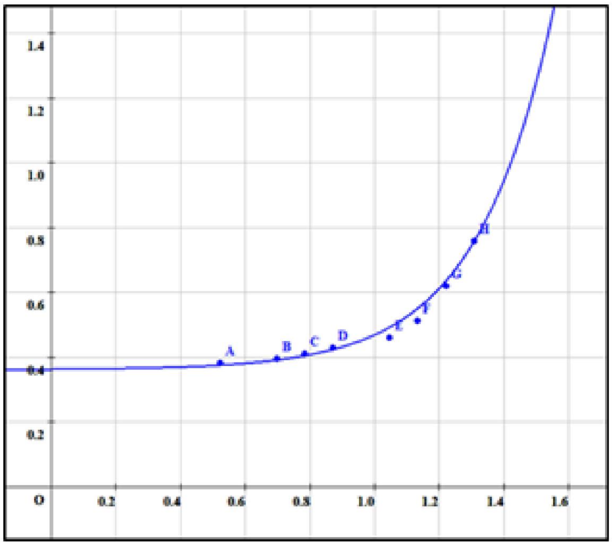
※2　概形(右上)・鉛直断面図(左下) / rough shape and its vertical cross section

表 2　実験 3 で用いた円錐の中心角及び概形

Table 2　Central angles and rough shapes of the cones used in the Experiment 3.

中心角 (central angle)	30°（A)	40°（B)	45°（C)	50°（D)
概形 (rough shape)				
中心角 (central angle)	60°（E)	65°（F)	70°（G)	75°（H)
概形 (rough shape)				

抵抗係数／Resistance coefficient



中心角/Central angle (rad)

・A~H support the points plotted in the Figure 1, respectively.

・The real line shows the estimated function in this study.

$$C_d=0.36+1.2\exp[4.2\{\theta-(\pi/2)\}]$$

(Therefore, C_d and θ are resistance coefficient and central angle, respectively.)

図 1　実験 3 によって得られた円錐の中心角と抵抗係数の関係(実線は本研究で推定した関数)

Fig 1　Relationship between central angle and resistance coefficient by the Experiment 3.