Description of Plasma Particle Motion by Quaternions

*Tadas Nakamura[1]

1. Fukui Prefectural University

Calculations of plasma particle motion using quaternions <gwmw class="ginger-module-highlighter-mistake-type-3" id="gwmw-158199789756932097994 38">has been reported</gwmw> at the SGEPSS fall meeting last year. It is known J.C. Maxwell used Hamilton's quaternions to express Maxwell equations in his masterpiece "Treatise on Electricity and Magnetism." The theory of vector analysis, which is familiar to us in modern physics, is developed a little after that by W. Gibbs and O. Heaviside. At the end of the 19th century and the beginning of the 20th century, there arises a heated controversy on this topic: which is a superior description of electromagnetism, the Gibbs/Heaviside's vector analysis or quaternions?

It turned out more and more physicists prefer the vector analysis and few people care quaternions in the latter half of the 20th century. However, quaternions resurrect in the 21th century in connection with computer graphics; the quaternion is found to be suitable to calculate 3 dimensional rotations in games or movies. Now more and more computer engineers learn the theory of quaternion for this purpose.

This fact suggests quaternions are suitable to express something related to rotations, whereas vectors are good for parallel transition. Provided that quaternions are suitable for rotations, it is expected that they are useful to express particle gyration in magnetized plasmas. The gyration in a 2 dimensional plane is often expressed by a complex exponential function in plasma physics. It can be extended to 3 dimensional case when we introduce quaternions to express rotations. The exponential function of quaternions becomes considerably complicated due to the their anti-commutable nature and we need to use Baker–Campbell–Hausdorff formula for multiplications. This complication is essential to express the complicated nature of 3 dimensional rotations.

<gwmw class="ginger-module-highlighter-mistake-type-3" id="gwmw-158199792005121124313 73">Actual calculation</gwmw> of the plasma curvature drift will be examined as an example in the talk. Usually the curvature drift is derived by replacing the centrifugal force <gwmw class="ginger-module-highlighter-mistake-type-3" id="gwmw-158199792083074167928 69">by</gwmw> gravitation, which is not straightforward. Using quaternions we can derive it from the basic equation of motion easily.

Keywords: Plasma Drift Motion, Magnetized Plasma, Quoternions

