A Multistate Low-dissipation Advection Upstream Splitting Method for Ideal Magnetohydrodynamics

*Takashi Minoshima[1], Keiichi Kitamura[2], Takahiro Miyoshi[3]

1. Center for Mathematical Science and Advanced Technology, Japan Agency for Marine-Earth Science and Technology, 2. Yokohama National University, 3. Hiroshima University

The magnetohydrodynamic (MHD) simulation is an indispensable way to study nonlinear dynamics in space and astrophysical plasmas. To capture high speed flows, discontinuities, and shocks that are frequently observed in these environments, many modern MHD simulation codes are built based on upwind schemes such as the Flux Vector Splitting method and the Flux Difference Splitting method. In particular, the HLLD approximate Riemann solver developed by Miyoshi and Kusano (2005) has an advantage of shock capturing capability with sufficient accuracy and robustness, and thus it becomes a de fact standard method for computational plasma physics.

The upwind schemes sometimes encounter severe numerical difficulties in stringent condition. The scheme tends to suffer from the numerical shock instability when a high Mach number multidimensional shock is well aligned to the grid spacing, and it causes a catastrophic solution such as the odd-even decoupling and “Carbuncle” phenomena. The mechanism of the instability has not been fully understood yet. On the other hand, the scheme is inherently hard to obtain a correct solution of very low Mach number (~0.01) flows, owing to excessive numerical diffusion. We need a novel numerical scheme to tackle a situation including both incompressible and hypersonic flows.

The Advection Upstream Splitting Method (AUSM; Liou et al.1993) and its variants (e.g., Liou 1996) have been widely adopted in computational aerodynamics. The AUSM-family schemes are alternative to other upwind schemes (FVS, FDS, HLL-type) in order to improve accuracy, robustness, and computational efficiency, and some of them are known to be robust against the numerical shock instability. Furthermore, recent AUSM-family schemes are extended to “all-speed” regime, in which a compressible simulation can get accurate solutions in low Mach number limit (Liou 2006, Shima and Kitamura 2011, Kitamura and Shima 2013). Although these advantages are quite attractive to MHD simulations as well, the extension of the scheme has been quite limited thus far (Han+09, Shen+12, Xisto+14, Kitamura+18).

Consequently, we propose a new AUSM-family scheme for MHD simulations. Following the AUSM methodology, we split the flux in MHD equations into the mass flux, pressure flux, and magnetic tension flux. The mass and pressure fluxes are designed to improve the robustness against the numerical shock instability and the accuracy of low Mach number MHD flows. The magnetic tension flux is built to be consistent with the HLLD solver so as to capture MHD discontinuities. The resulting scheme, terms Multistate Low-dissipation AUSM, is expected to overcome numerical difficulties inherent in familiar upwind schemes. We present several benchmark tests including typical shock tube problems, nearly incompressible Kelvin-Helmholtz instability, and Richtmyer-Meshkov instability to verify the expected capability. Hence, the scheme must be a promising tool to tackle the solar-terrestrial system that includes low speed flows (e.g., solar interior and surface) as well as high speed flows (e.g., solar wind and CMEs).

Keywords: magnetohydrodynamics, shocks, shear flow, magnetic fields, all-speed scheme

