Reduction of mathematical model for slip behavior of one dimensional elastic layer with rate and state friction

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It is already accepted notion that slow earthquake occurs at the region between steady sliding and seismogenic zones[1]. Inspired from such observation, in this study, we explore the mathematical model of one-dimensional thin elastic layer incorporating a rate and state friction model, especially focused near the Hopf-bifurcation point. Hopf-bifurcation is a bifurcation from steady sliding state to oscillatory state. Intuitively, the behavior near such bifurcation point should be related with slow earthquake which occurs at the region between steady sliding to seismogenic zones.

Our model starts from one-dimensional thin elastic layer sheared between two parallel solid plates which moves perpendicular to the spatial dimension at the constant relative velocity. This situation is reduced to be one-dimensinal wave equation incorporating the effect of the friction at the bottom plate[2]. As a friction, we used rate and state friction model, which is introduced by Dietrich[3]. These situation lead to partial differential equations with three variables, displacement, speed and state. Despite of extremely simplified situation, there still remains ten parameters to be determined. However, our concerns are limited to the behaviors near the Hopf bifurcation point of our mathematical model. By focusing near the bifurcation point, we can reduce the equation into complex Ginzburg Landau equation, which has only one complex variable with three complex parameters. The reduced equation also predicts that the model exhibits spatio-temporal chaos due to Benjamin-Feir instability as shown in Fig. (a) by the spatio-temporal diagram. By applying the threshold value of the slip velocity, we can discretize the obtained slip data as in Fig. (b). In this way, we found that the statics of the slip shows exponential distribution in the spatio-temporal chaos regime as in Fig. (c).

The size of such slip can be also deduced from physical parameters analytically. Some engineering circumstances and room analogue experiments, we can show that the current model can be applied successfully. Unfortunately, however, our prediction, the duration and the spatial size of slip, are NOT consistent with geophysical observations. Thus, we conclude that slow earthquake can NOT be simplified as spatio-temporal chaos observed in one dimensional elastic layer. In this talk, we will show the detail of the mathematical model, as well as the result of reduction. In addition, we show the observed spatio-temporal chaos regime and their inconsistency with geophysical observation.

[Reference]
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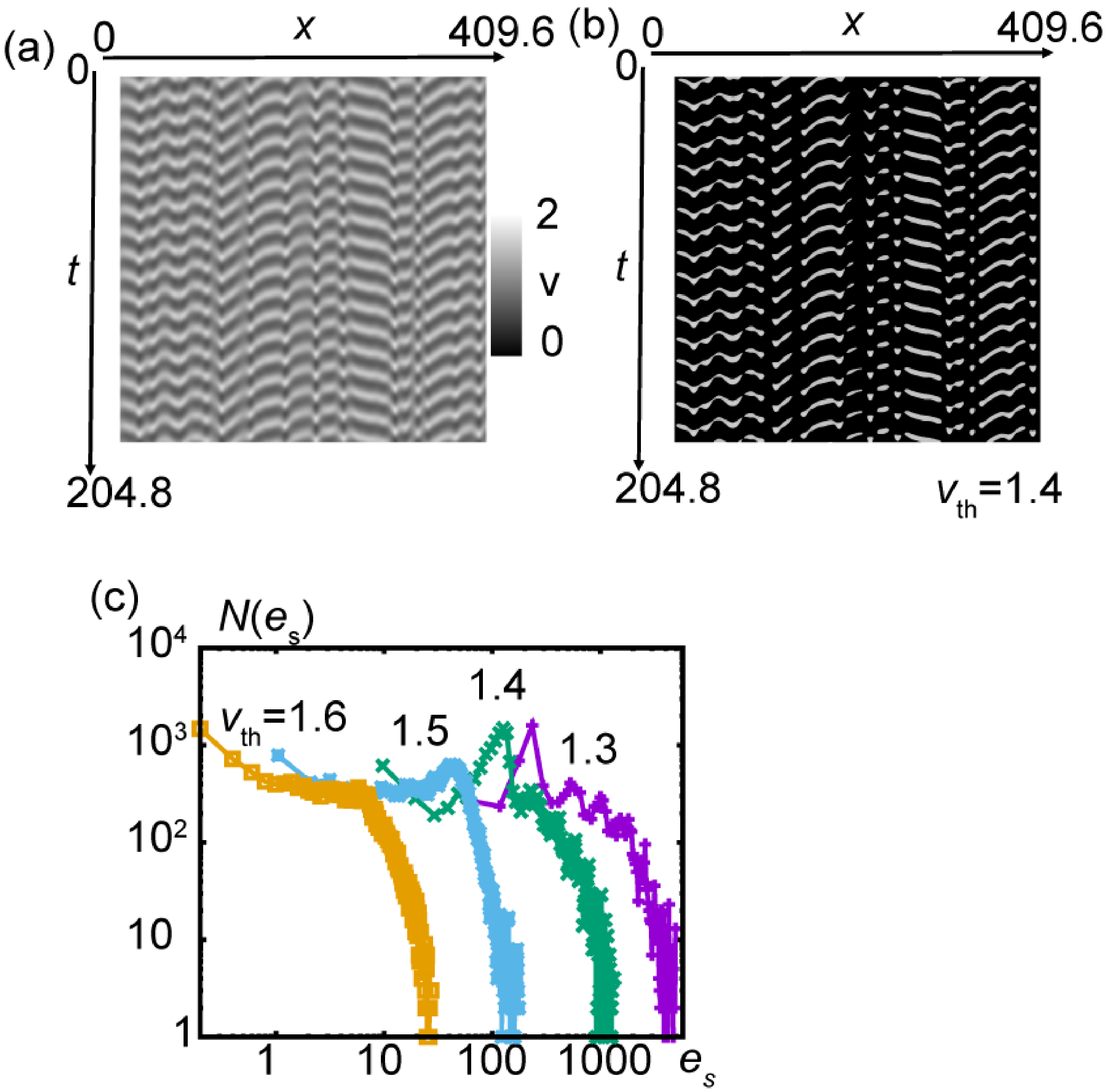
(a)
0
x
409.6
0
t
2
v
0
204.8
(b)
0
x
409.6
0
t
204.8
v_{th}=1.4
(c)
$N(e_s)$
10^4
10^3
10^2
10
1
v_{th}=1.6
1.5
1.4
1.3
1
10
100
1000
e_s