Decomposition of elastic potential energy and the meaning of the energetics-based failure stress criterion

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Recently, we proposed an energetics-based failure stress (EFS) and demonstrated that its change (ΔEFS) associated with a main shock gives a new rational metric for evaluating aftershock generation (Terakawa, Matsu'ura & Noda, EPSL, 2020). The EFS is defined in a similar form to the well-known Coulomb failure stress (CFS = shear stress −fault strength), but the shear stress is replaced with the square root of the second invariant of deviatoric stress tensor, which is proportional to the shear strain energy density. The expression of ΔEFS can be reduced to that of ΔCFS in a special case, but the meaning of ΔEFS is essentially different from that of ΔCFS. In this presentation, we make clear the rationale and meaning of EFS through theoretical considerations on the elastic potential energy density and the decomposition of elastic stiffness tensor.

The potential energy density U of linear elastic forces is generally represented in the quadratic form of strain tensor components ε_{ij} as $U = C_{ijkl}\varepsilon_{ij}\varepsilon_{kl}$, where the fourth-order tensor C_{ijkl} denotes the elastic stiffness. Then, by definition, the stress tensor components σ_{ij} are given by the partial derivative of U with respect to ε_{ij} as $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$. When the medium is isotropic, the stiffness tensor C_{ijkl} is expressible as a linear combination of two independent symmetric tensors (Walpole, Proc. R. Soc. Lond., 1984), and so the elastic potential energy density U can be decomposed into two independent parts; the volumetric part U_v and the shearing part U_s (Matsu'ura, Noda & Terakawa, Tectonophysics, 2019). In such a case, the partial derivative of U_v with respect to volumetric deformation (the first invariant of strain tensor) gives the mean normal stress (one-third of the first invariant of stress tensor). Similarly, the partial derivative of U_s with respect to shearing deformation (the square root of twice the second invariant of deviatoric strain tensor) gives the shear stress (the square root of twice the second invariant of deviatoric stress tensor). These generalized forces to volumetric and shearing deformations in Lagrangian mechanics are used for constructing the energetics-based failure stress EFS.

The elastic property of the Earth's crust is not necessarily isotropic, and so, strictly speaking, we may not decompose the elastic potential energy density into two independent parts. However, even in such a case, it is possible to define the closest isotropic fourth-order tensor to a given anisotropic stiffness tensor (Moakher, Quart. J. Mech. Appl. Math., 2008). Using the closest isotropic fourth-order tensor, we can decompose the elastic potential energy density into two independent parts, and define the generalized forces needed for constructing the EFS in a similar way to the isotropic case.

The expression of ΔEFS, which is a metric for evaluating aftershock generation, can be reduced to the maximum change in shear stress (the square root of the second invariant of deviatoric stress change tensor) recommended by DeVries et al. (Nature, 2018), when the level of background deviatoric stress is zero and the coseismic changes in confining pressure and pore-fluid pressure are negligible. On the other hand, when the level of background deviatoric stress is much higher than the maximum change in stress, it can be reduced to the ΔCFS on the assumption that both the background deviatoric stress and the coseismic deviatoric stress change are in the state of pure shear. From an energetic point of view, however, there is not much meaning in doing such an interpretation of ΔEFS , because the shear stress in the definition of EFS is the generalized force to shearing deformation, and its coseismic change is simply evaluated by the inner product of the deviatoric stress change tensor and the normalized background deviatoric stress tensor, which can be estimated from the CMT data inversion of background seismic events (Terakawa & Matsu'ura, GJI, 2008; Tectonics, 2010).

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