Shallow water MHD waves trapped near the poles in a stably stratified outermost Earth's core

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Magnetohydrodynamic (MHD) shallow water linear waves are investigated over a rotating sphere with an imposed equatorially antisymmetric toroidal magnetic field: $B_{0\Phi}=B_0\sin\theta\cos\theta$, where B_0 is a constant, θ is the colatitude and Φ is the azimuth. This system can imitatively represent the dynamics of a liquid metal within a stably stratified layer at the top of the Earth's core, which was detected through seismological surveys (e.g. Helffrich & Kaneshima, 2010[1]) and also has been deduced from geophysical and geochemical knowledge (e.g. Buffett & Seagle, 2010[2]; Pozzo et al., 2012[3]; Gubbins & Davies, 2013[4]; Brodholt & Badro, 2017[5]). Because slowly propagating waves in the liquid core can result in geomagnetic secular variations, comparison between exhaustive studies of MHD waves in a rotating stratified fluid and observations of geomagnetic fluctuations should provide constraints on the obscure stratified layer in the outermost core (e.g. Braginsky, 1993[6]; Buffett, 2014[7]).

The adopted configuration of the background field complicates solving the eigenvalue problem of linear waves due to the emergence of an Alfvén continuum and critical latitudes unless dissipation effects are taken into account. These result from non-dissipative Alfvén resonance, which occurs only when $B_{0\Phi}/\sin\theta$ depends on θ, that is, regular singular points appear in the differential equation of linear problems. The solutions of the continuum are required to express the transient evolution of an arbitrary initial disturbance (e.g. Case, 1960[8]; Goedbloed & Poedts, 2004[9]). We can confirm numerically and analytically that introducing magnetic diffusion eliminates these Alfvén continuous modes and their singular structures around critical latitudes (Nakashima, Ph.D. thesis, 2020[10]).

For the Earth's core-like parameter ($B_0\simeq0.5$—5mT and magnetic diffusivity $\eta\simeq1\mathrm{m}^2/\mathrm{s}$), westward polar trapped modes are obtained as eigenmodes, which have a period of around from several to 1000 years. We may be able to observe these modes as geomagnetic secular variations in high latitude regions, if the strength of stratification in the stratified layer is close to the estimate of Buffett (2014)[7]. The analyses of recent geomagnetic models and paleomagnetic data in terms of such waves could confirm the robustness of previous estimates of the properties of the layer.

[Reference]

[1] Helffrich, G., Kaneshima, S. (2010) *Nature*, **468**, 807. doi: 10.1038/nature09636
[2] Buffett, B. A., Seagle, C. T. (2010) *J. Geophys. Res.*, **115**, B04407. doi: 10.1029/2009JB006751
[3] Pozzo, M., Davies, C., Gubbins, D., Alfè, D. (2012) *Nature*, **485**, 355. doi: 10.1038/nature11031
[4] Gubbins, D., Davies, C. J. (2013) *Phys. Earth Planet. Inter.*, **215**, 21. doi: 10.1016/j.pepi.2012.11.001
[5] Brodholt, J., Badro, J. (2017) *Geophys. Res. Lett.*, **44**, 8303. doi: 10.1002/2017GL074261
[6] Braginsky, S. I. (1993) *J. Geomag. Geoelectr.*, **45**, 1517. doi: 10.5636/jgg.45.1517
[7] Buffett, B. (2014) *Nature*, **507**, 484. doi: 10.1038/nature13122
[8] Case, K. M. (1960) *Phys. Fluids*, **3**, 143. doi: 10.1063/1.1706010
[9] Goedbloed, J. P., Poedts, S. (2004) *Principles of magnetohydrodynamics: with applications to laboratory and astrophysical plasmas,* Cambridge Univ. Press, Cambridge.
[10] Nakashima, R. (2020) Ph.D. thesis, Kyushu University.

http://dyna.geo.kyushu-u.ac.jp/HomePage/nakashima/pdf/doctoral_thesis.pdf

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