Towards application of data-driven sparse sensor placement technique developed in fluid mechanics to seismology

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Reduced-order modeling gathers a lot of attention in fluid mechanics for fluid analysis and flow control. Proper orthogonal decomposition (POD)[1] is one of the effective methods to decompose the high-dimensional fluid data into significant modes. The dimension of the flow field data can be reduced by reconstructing the data using several selected significant modes. In addition, if the data can be effectively expressed by a limited number of POD modes as shown in Fig. 1, the limited sensor placed at appropriate positions gives us the approximated full state information as follows[2]:

$\boldsymbol{y} = \boldsymbol{HUx}$, (1)

where $\boldsymbol{y} \in \mathbf{R}^{p}$, $\boldsymbol{H} \in \mathbf{R}^{p\times n}$, $\boldsymbol{U} \in \mathbf{R}^{n\times r}$, and $\boldsymbol{x} \in \mathbf{R}^{r}$ are the observation vector, the sparse sensor location matrix, the spatial POD modes, and the POD mode amplitude, respectively. Here, p, n, and r are the number of the sensor location, degree of freedom of the spatial POD modes, and the rank for truncated POD, respectively.

Schematic diagram of Eq. (1) is illustrated in Fig. 2. This problem is considered to be one of the sensor selection problems when the POD mode $\boldsymbol{U}$ and the strength x are assumed to be a sensor-candidate matrix and the latent state variables, respectively. Obtaining $\boldsymbol{H}$ matrix corresponds to the computation for the sensor selection, and the convex approximation method[3] and the QR-based greedy algorithm[2] have been used. In our group, data-driven sparse sensor selection algorithms for the data obtained in experimental fluid dynamics have been developed. The data in the fluid dynamics is noisy high-dimensional multi-component data so that a fast and robust sensor selection algorithm is necessary to realize a real-time on-line measurement and flow control. We proposed three algorithms recently.

Fast Greedy sensor selection algorithm for the scalar data

The computational cost is a critical problem for high-dimensional data analysis. We extended the previous QR-based greedy algorithm and reduced the computational cost. In our method, the objective function of the problem was redefined to be the maximization of the determinant of the matrix appearing in pseudo-inverse matrix operation. The procedure for the maximization of the determinant of the corresponding matrix is the same as that of the previous QR-based greedy method when the number of sensors less is less than or equal to that of state variables. When the number of sensors greater than that of state variables, new sensors are calculated by a proposed determinant-based greedy method which is accelerated by both determinant formula and matrix inversion lemma.

Fast Greedy sensor selection algorithm for the vector data

The multi-component data such as velocity distributions in x and y directions frequently appear in the fluid data. The sparse sensor selection using the convex approximation algorithm for the vector data was valid, but it requires long computational time to analyze high-dimensional data due to its expensive computational cost. Therefore, we extended our determinant-based greedy algorithm, which is a much lower computational cost compared to convex approximation, for vector data.

Fast greedy sensor selection in measurement with correlated noise

The noise-robust greedy sparse-sensor-selection method based on the determinant maximization was proposed. Optimization is conducted by maximizing the determinant of the matrix which corresponds to the inverse matrix appearing in the Bayesian estimation operator. The computational cost is slightly larger than that of the previous methods, but the estimation accuracy is much improved. We will apply and improve these methods to seismic research with the seismic research group.

References

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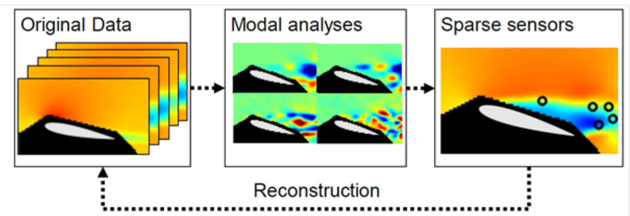

Figure 1 Schematic diagram of the present study.

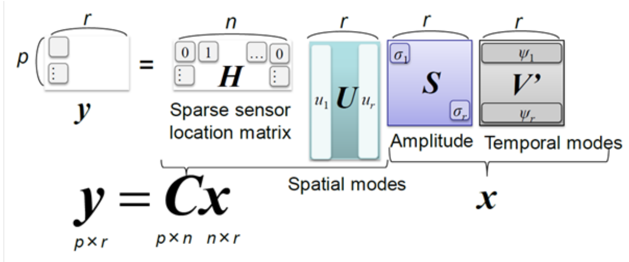

Figure 2 Schematic diagram of the sensor placement problem.