Causes and Solutions of Computational Instability in Tsunami Calculations by Finite Difference Method for Nonlinear Long Wave Equations

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When solving the nonlinear long wave equation by the finite difference method, which is commonly used in tsunami numerical simulations. The calculation may become unstable and the results may diverge even though the CFL condition is satisfied. When considering the issuance and updating of tsunami warnings using real-time simulations, such calculation failures can be fatal. Therefore, in order to eliminate such calculation instability, the evaluation of the difference equation and the calculation condition were examined, and the method to prevent calculation instability was verified.
First of all, the author has been examining the difference of time difference equation of friction term in nonlinear long wave equation (MINAMI 2020JpGU etc.), and examined the discretization error and computational stability of four difference methods, (1)Explicit (2)Semi-implicit(combined) (3)Semi-implicit(simple) (4)Full-implicit. Among them, Semi-implicit (simple) and Full-implicit are better in computational stability. However, many of the current tsunami models use Semi-implicit (combined) with various conditions. Therefore, in this paper, we evaluate the computational conditions in the TUNAMI-N2 model (Imamura et al. 2006, etc.).
In the TUNAMI-N2 model, Semi-implicit (combined) is used, and the following four conditions are added to the calculation: 1. If the total water depth is less than 0.1 cm before the flux calculation, the flux for that time step is set to zero. 2. If the total water depth is less than 1 cm when calculating the friction term, it is rounded up to 1 cm. 3. When the total water depth is less than 0.1cm, the advection term is not calculated (even when the total water depth is more than 0.1cm, only the upwind term is not calculated if the total water depth on the upwind side is less than 0.1cm). 4. If the calculated flux is smaller than 10^-10m, set the flux to zero.
As a result, even with the above conditions, there were cases where the semi-implicit (combined) produced large errors in the calculations, as shown in Fig. 1. This is a case where the flux converges to near zero when solved continuously, but at this time, due to the difference in the way the friction term is differentiated, the flux does not converge to zero and the positive and negative directions are reversed. When this inversion of the flux direction occurs, the upwind direction of the upwind difference is switched when calculating the advection term, and the upwind value that should be referenced is switched. This was found to be the cause of the time evolution and enlargement of the error. It is possible to avoid these problems by changing the numerical values of the above conditions, but this means that the advection term and the friction term disappear when the water depth is small, which also increases the error.
In addition, the instability of the calculation at the edge of the computational domain was also verified. In the finite difference method for tsunami computation, the method using the characteristic curve of progressive long wave (Goto and Ogawa 1982) is often adopted. This method assumes linear long waves, and if there is a shallow water mesh at the edge of the computational domain, the influence of friction and advection terms increases and the error becomes large. Therefore, frictional forces were also considered in the calculation at the boundary, the error was found to be small (Fig. 2). The water depth in the experiment is uniform, and the results are compared with the calculation using the periodic boundary condition.
From the above verification, it was confirmed that when solving the nonlinear long wave equation by the

finite difference method, it is possible to perform more stable calculations than before by using a more implicit difference equation for the friction term and by performing more appropriate processing at the edge of the calculation domain.

Keywords: Tsunami Calculations, Computational Instability, Nonlinear Long Wave Equations

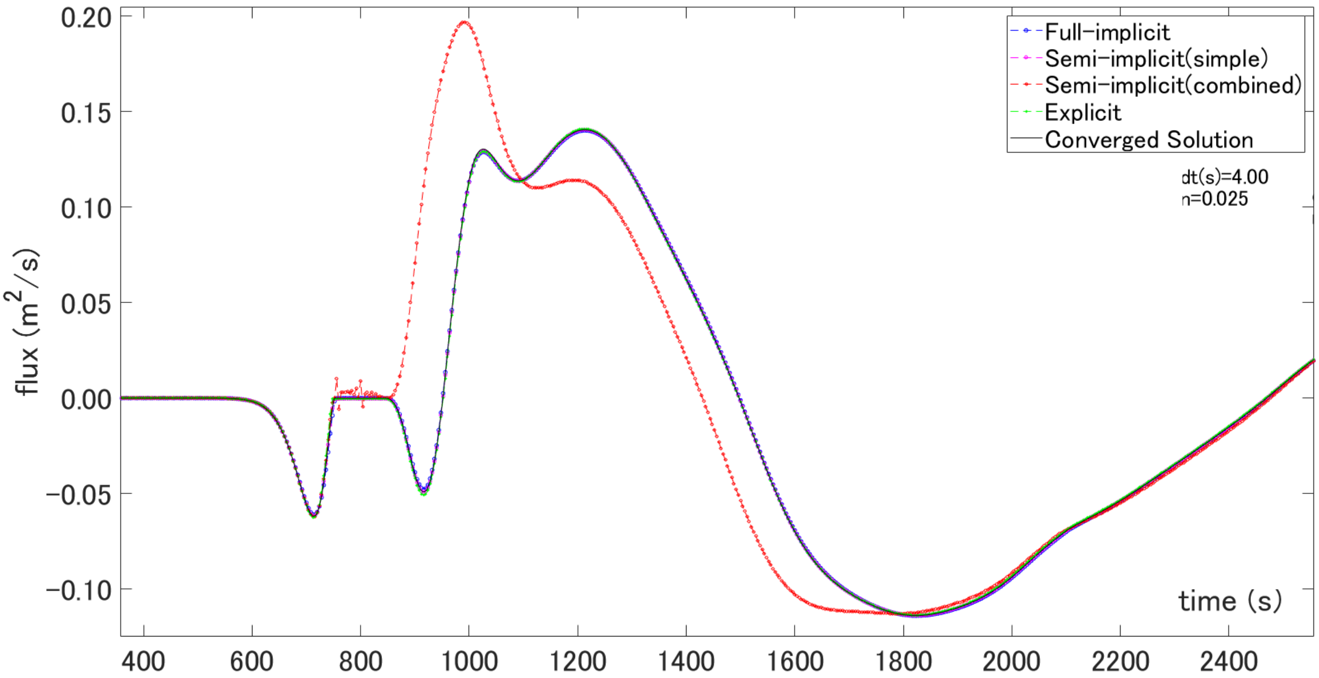

Fig 1. Case of instability using Semi-implicit(combined)

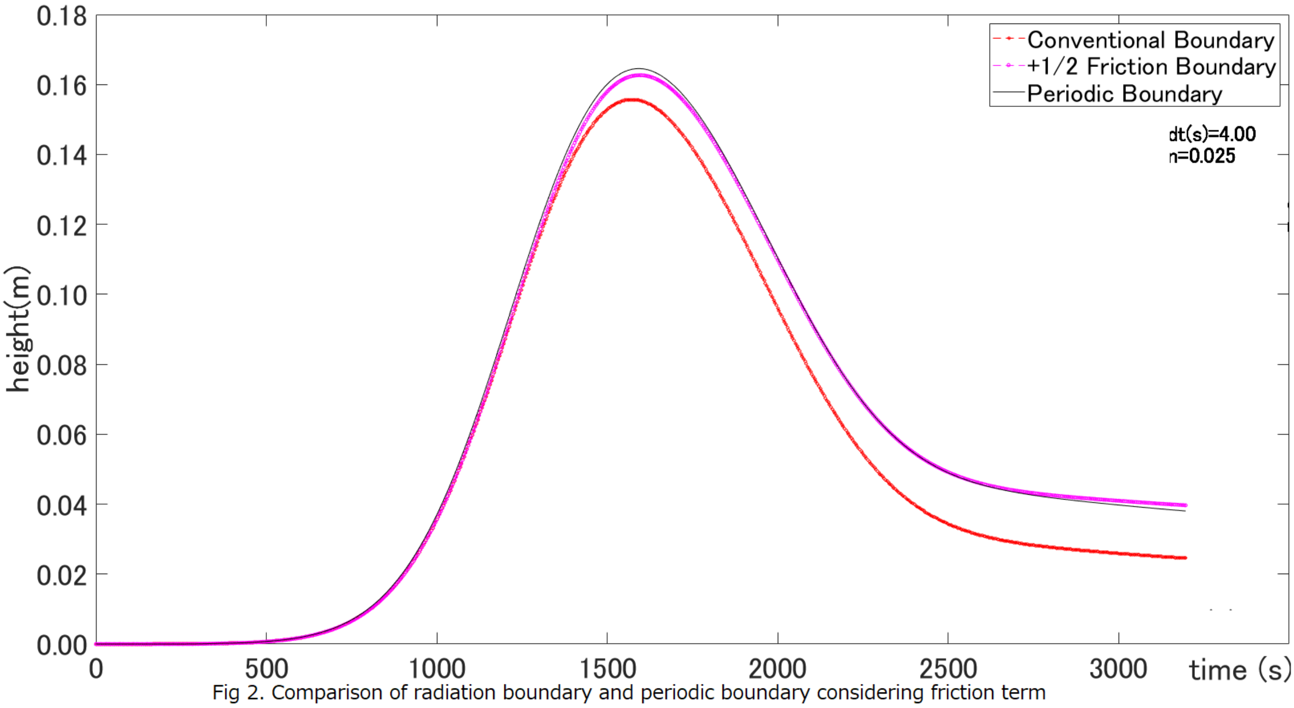

Fig 2. Comparison of radiation boundary and periodic boundary considering friction term