A shallow-water model of gravity currents for a wide range of density differences and slope angles: Toward a time-dependent two-dimensional two-layer model of pyroclastic density currents

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Various gravity currents such as turbidity current, debris flow, and pyroclastic density current (PDC) occur on the Earth's surface. The most fundamental parameter that characterizes the dynamics of gravity currents is the ratio of the current density (ρ) to the ambient fluid density (ρ_a). The currents with $\rho / \rho_a = 10^0–10^1$ (e.g., turbidity currents; the upper dilute region of PDCs) form a thick, slow, bulbous-shaped head at the flow front due to the resistance of ambient fluid, whereas the currents with $\rho / \rho_a \geqq 10^2$ (e.g., debris flows; the lower dense region of PDCs) form a thin, fast, wedge-shaped head because the momentum loss due to the ambient resistance is negligibly small. Previous one-dimensional (1D) shallow-water models to reproduce such effects of ρ / ρ_a consist not only of the conservation equations of mass and momentum to obtain the flow thickness $h(x,t)$ and flow velocity $u(x,t)$ at a position x and a time t but also of a moving boundary condition at the flow front (i.e., the front condition) to obtain the front position $x_N(t)$. The front condition assumes the quasi-steady mechanical balance between the resistance pressure of the ambient fluid and the driving hydrostatic pressure of the current. Although the previous models have been widely applied to 1D models of turbidity currents and PDCs, they (i.e., the front condition) may not be applied to geophysical situations where a gravity current flows over complex topography. Even if the previous 1D models are extended to 2D models to evaluate the effect of complex topography, numerical difficulties arise in calculating the front condition in 2D space. This study proposes a new shallow-water model where the ambient fluid resistance is modeled directly in the momentum conservation equation instead of by imposing the front condition.

Whereas the previous models consider the momentum loss due to the ambient resistance only at the flow front assuming the quasi-steadiness, the new model accounts for it both at the front and behind the front without the quasi-steady assumption. To investigate the difference in the results between the new and previous models, these models are applied to the 1D dam-break problems for $\rho / \rho_a = 2$ and slopes angles (1) 0° (flat surface), (2) -10° (upslope), and (3) 10° (downslope). For (1) the flat-surface case, the new-model results are fitted to the previous-model results, since the previous model has been validated by comparisons with many laboratory and numerical experiments. When the coefficient of ambient resistance in the new model (C_d) is set to 0.4, the new-model results largely agree with the previous-model results. The new-model results have slightly smaller velocity and slightly larger thickness than those of the previous model, which is due to the momentum loss caused by the ambient resistance behind the front. (2) For the upslope case, the new-model results for $C_d = 0.4$ largely agree with the previous-model results at the beginning of the propagation. However, after the momentum loss due to the upslope reduces the frontal velocity to zero, the new-model results deviate from the previous-model results. The new model can reproduce the subsequent behavior from backflow to standstill, whereas the previous model cannot be applied to such a complicated situation. (3) For the downslope case, the new-model results for $C_d = 0.4$ also largely agree with the previous-model results at the beginning of the propagation. Subsequently, however, only the new-model results show that an increase in thickness occurs at the point with $\partial h/\partial x < 0$ just behind the frontal head, which is due to the increase in the ambient resistance on the flow top with $\partial h/\partial x < 0$ and large u. Such a phenomenon has been observed in previous 3D numerical experiments, which can be explained by the new model. In the future, we will develop a time-dependent 2D two-layer PDC model based on the new model and evaluate the topographic effects on the dynamics of PDCs (especially, their upper dilute region).

Keywords: Gravity current, Shallow-water model, Density ratio, Slope angle, Resistance of ambient fluid, Frontal boundary condition