

# Inversion of envelope widths of volcanic earthquakes to image scattering structures using space-weighting functions

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The envelope widths of natural and active seismic sources have been used to estimate one-dimensional (1D) layered seismic scattering and attenuation structures at volcanoes, which indicate that a surficial, highly heterogeneous and attenuative layer up to around 1 km thickness commonly exists. The envelope width ( $p$ ) is defined by the ratio of the cumulative to peak amplitudes for S waves in high-frequency (5–10 Hz) envelope seismograms. In this study, we developed an inversion method to estimate spatial distributions of the scattering mean free path ( $l_0$ ) and the quality factor for medium attenuation ( $Q_i$ ) using the residuals between observed  $p$  values and those calculated with 1D  $l_0$  and  $Q_i$  models. One of the difficulties in performing such an inversion is the computational cost to calculate the partial derivatives of the envelope widths with respect to  $l_0$  and  $Q_i$  perturbations in individual grids for all combinations of source-station pairs to construct the data kernel. To overcome this difficulty, we proposed an efficient approach to calculate the partial derivatives using the 3D space-weighting functions (SWFs) for the 1D models. In our inversion, we used the quality factor for scattering attenuation ( $Q_s$ ) instead of  $l_0$ , and focused on the first surficial layer because the envelope width is insensitive to the second layer. We first calculated the energy distributions in the individual 3D grids for the 1D models by Monte Carlo envelope simulations for individual source locations. The resultant 3D energy distribution for each source location is regarded as the SWF. Then, we calculated the partial derivatives with respect to  $Q_s^{-1}$  and  $Q_i^{-1}$  perturbations in the entire first layer for all combinations of source-station pairs. These partial derivatives were weighted by the SWF to obtain the partial derivatives in the individual grids for each source-station pair, which were used to construct the data kernel. We solved the damped least-squares problem for the envelope width residuals with the hyper-parameters for  $Q_s^{-1}$  and  $Q_i^{-1}$  corrections in the individual grids. Our tests of this inversion method with synthetic envelope waveforms indicated that iterative inversions are required to suppress artifacts tended to appear beneath station locations. Our inversion results for the observed envelope width data at Taal (Philippines), Nevado del Ruiz, and Galeras (Colombia) indicate that there exist smaller  $l_0$  and larger  $Q_i$  areas near the summit craters at all these volcanoes, which may correspond to heterogeneous, unconsolidated sediments with cracks filled with steam in volcanic conduits. Smaller  $Q_i$  areas found beneath the eastern flank at Taal matched with an S-wave attenuation region, where the existence of actively degassing magma was inferred by a previous study. These results demonstrate that our inversion method using the envelope widths is useful to characterize spatial distributions of scattering and attenuation structures at volcanoes.