Dynamic displacement, strain and stress fields associated with polygonal and polyhedral sources in a three-dimensional elastic space: An elastodynamically extended form of the Burgers displacement equation

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This study presents an analytic regularised discretised set of displacement-, displacement-rate-, strain- and stress-nuclei in the three-dimensional elastodynamic problems of a homogeneous isotropic finite space. It covers all the deformation sources, namely, boundary displacement rate, displacement-gap rate, boundary traction and volume sources, by interpolating them with polygonal boundary and polyhedral volume elements in a spatially piecewise-constant and temporally piecewise-polynomial manner.

First, a simplified form of the representation theorem for isotropic homogeneous elastostatic crack problems, known as the Burgers displacement equation, is extended to an elastodynamic form. There, spatial piecewise-constant interpolation on a boundary element allows us to decompose the boundary integral to two line integrals along the side of the boundary element and along the wavefront projected onto the element. A line-integral representation of the traction boundary integral is also shown for planar boundaries. Second, the volume integral is obtained using the spatially piecewise-constant potential fields of the volume sources. Such interpolation utilising the Helmholtz decomposition of the volume sources maps the volume integral of an arbitrary volume source to the boundary integrals of eigentraction on surfaces of the volume source, and it unifies the polyhedral volume integrals with polygonal boundary integrals in the form of the line integrals. Third, their integral calculation is shown for polygonal boundary and polyhedra with temporal piecewise-polynomial interpolation of any degrees. We investigate the numerical usability of the obtained expressions free from any artificial singularities, and examined codes are distributed as supplements. Present discredited boundary and volume integrals constitute a dynamic counterpart to the classic dislocation theory of static Burgers vectors.

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