Fractal Size Reduction and Critical Slip Displacement during Fault Slip

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1. Introduction
To evaluate an occurrence of unstable slip, a systematic understanding of frictional instability is necessary. Critical slip displacement has been used to evaluate frictional instability of faults. The critical slip displacement is defined as the slip over which strength breaks down during fault slip or the slip distance, at a constant velocity, through which a population of contacts is destroyed and replaced by an uncorrelated set [Marone and Kilgore, 1994; Scholz, 2002]. Although previous studies have proposed various relationships between critical slip displacement and other factors [e.g., Sammis and Biegel, 1989; Marone and Cox, 1994], it is still open to debate that which factors dictate the critical slip displacement. In this study, we aim to theoretically clarify a factor that determines the critical slip displacement. Additionally, as fractured areas have a self-similar structure, we utilize fractal theory for understanding of the critical slip displacement. Through this study, it is clarified that comminution characterized by the fractal size reduction determines the critical slip displacement.

2. Fractal dimensions with comminution processes
A fractal dimension of surface roughness changes with comminution processes of materials: It takes on the values 3.0, 2.5, and 2.0 from high-velocity impact experiments, conventional milling, and finer grinding, respectively [Hukki, 1962; Rumpf, 1973; Nagahama, 1991, 1993; Muto et al., 2015]. Let us consider the underlying cause of changing in the fractal dimension of surface roughness. When rocks receive external force, a portion of work done externally is used for energy dissipation for fracturing. The energy per unit mass for fracturing is in proportion to the exponential function of a characteristic linear dimension with the fractal dimension of surface roughness as an exponent. Consequently, comminution processes influence the fractal dimension of surface roughness as well as dissipative energy for fracturing.

3. Relationship between critical slip displacement and shear strain
Shear strain for crushing gouge is defined as a ratio of a function related to particle size to shear stress [Draper, 1976]. The function can be approximately expressed as the maximum grain diameters after shearing with the fractal dimension as an exponent. The critical slip displacement is approximately proportional to the final maximum grain size [Dieterich, 1981]. Therefore, the relationship between the critical slip displacement and shear strain can be described as a linear function on a log-log plot. The slope of this function is related to the fractal dimension of the surface roughness. This relationship obtained from theoretical analysis is consistent with previous experimental datasets [Marone and Kilgore, 1993]. Thus, the fractal dimension of the surface roughness controlled determines the critical slip displacement.

4. Discussion –implication of dissipative energy
Fractal size reduction of materials also determined the surface roughness and particle size ranges. Through the theoretical analysis, it is clear that large dissipative energy means small critical slip displacement and/or high fractal dimension of surface roughness. The fractal dimension of the surface roughness ranges from two for smooth surfaces to almost three for rough surfaces [Avnir et al., 1983; Nagahama, 1993]. Thus, small fractal dimension of surface roughness, or small dissipative energy, indicates that gouge particles with various size ranges compose smooth surfaces with filled opening areas.

5. Summary
From this study, it is obvious that the critical slip displacement is originally determined by fractal size reduction of materials. Difference in comminution processes produces the differences in dissipative energy for fracturing, particle size ranges, the surface roughness, and the critical slip displacement.

Keywords: critical slip displacement, dissipative energy, frictional instability , fractal