A generalized equation for the resonance frequencies of a fluid-filled crack

*Yuta Maeda[1], Hiroyuki Kumagai[1]

1. Nagoya University

Although a model of the resonance of a rectangular fluid-filled crack (crack model; Chouet, 1986, JGR) is one of the most frequently used source models of long-period (LP) seismic events at volcanoes, there has been no analytical solution for the resonance frequencies. We previously proposed an empirical expression for the resonance frequencies (Maeda and Kumagai, 2013, GRL):

$f_m = (m - 1)a/\{2L_x[1 + 2\varepsilon_m C_x]^{1/2}\}$, (1)

where a is the sound velocity of the fluid, L_x is the crack length along the wave propagation direction, m is the mode number defined such that the wavelength is $2L_x/m$, C_x is the crack stiffness, and ε_m is an empirical constant that depends on the crack aspect ratio χ and oscillation mode m. Although eq. (1) can potentially be used to compute the resonance frequencies easily, the requirement to determine the value of ε_m numerically for each crack aspect ratio and oscillation mode has prevented widespread use of the equation for interpretations of LP events at volcanoes.

In the present study, we examined the theoretical basis for the expression. We assumed that the ratio of the crack wall displacement to the fluid pressure near each crack edge varied as the square root of the distance from the edge. Using this assumption, we showed theoretically that eq. (1) was a good approximation (difference 2%) to another more complete expression:

$f_m = (m - 1)a/(2L_x I_m)$, (2)

$I_m = (1 - 4\gamma/5m)J_m(g_{m0}C_x) + (16\gamma/15m)[1/K_m(g_{m0}C_x) + 1/K_m(g_{m0}C_x)^2]$, (3)

where $J_m(\xi) = (1 + 2\xi)^{1/2}$, $K_m(\xi) = J_m(\xi) + 1$, $\gamma = 0.22$, and

$g_{m0} = (1 - 4\gamma/3m\chi)/(3m - 4\gamma)$ (4)

for $\chi > 4\gamma/m$ and

$g_{m0} = (2/3)(m\chi/4\gamma)^{1/2}/(3m - 4\gamma)$ (5)

for $\chi < 4\gamma/m$. The constant g_{m0} in eqs (4) and (5) is related to ε_m in eq. (1) as $\varepsilon_m = g_{m0}(3m - 4\gamma)/(3m)$.

This theoretical expression (eqs 2-5) is a closed form of a mathematical function of the crack model parameters and oscillation mode number; there are no empirical constants to be determined numerically. The expression thus enabled us to analytically compute the resonance frequencies for arbitrary rectangular cracks, and the results were in good agreement (difference 5%) with numerical solutions. Resonance frequencies of cracks can be very easily predicted using this expression. This predictive ability may enhance our quantitative understanding of the processes that generate LP events at volcanoes.

Keywords: LP event, Resonance, Crack

