Singular-Value Analyses of Perturbations on a Cylindrical Vortex Sheet

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A cylindrical vortex sheet is an idealized vortex consisting of a cylinder of infinite vorticity, whose thickness is zero, and zero vorticity regions elsewhere. It is often adopted as a model for the explanation of a multiple vortex structure seen in tornado-like vortices. There, the cylindrical vortex sheet and the perturbations developing on it are regarded as a parent vortex and the secondary vortices formed within it, respectively, and linear stability of cylindrical vortex sheet against such symmetric and/or asymmetric perturbations is investigated. Since the analyzing field is zero vorticity except for the cylindrical vortex sheet of infinitesimal thickness, Bernoulli's equation is used as the governing equation.
Here, it is necessary to carry out a singular-value analysis, not an eigenvalue analysis, to detect a possible perturbation which grows most rapidly during a given finite target time. This is because such an optimally-excited perturbation is, even if the equation is linear, not necessarily the most unstable eigen mode due to a non-normality of the governing equation. In this study, therefore, a singular-value analysis is carried out instead of an eigenvalue analysis. Three norms (i.e. L2 norm, energy norm, and Sobolev norm) are adopted as indices to measure the growth of perturbations, and then analytical forms of singular-values are derived for each norm. As a result of the analyses, it is revealed that non-normal growth of the perturbations is seen in all parameter space, where the amplitude of perturbations under L2 norm is generally largest and that under energy norm is the minimum.

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