Propertime Path Integral for Relativistic Diffusion

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It is well known that there exist infinite speed components in a solution of a simple diffusion equation with the first order time derivative. This does not cause serious problems if that part is small enough in non-relativistic regime, however, it may cause spurious growing solution in relativity. The reason is that propagation faster than the speed of light means backward propagation in time in some reference frame, and time reversal diffusion equations may have growing component.

This difficulty is inevitable for equations with first order time derivative, and hence, equations with second order have been proposed by Israel and Stewart (1970) for relativistic thermodynamics. Theories in this line are called "causal thermodynamics" and have been extensively studied since then. Second or higher order time derivative can make the propagation speed under a certain finite value to avoid the non-causal propagation. However, these higher order terms are not based on some physical reasoning of underlying mechanism; they are mathematical device to avoid infinite speed. Solutions of these equations do not violate causality, but it does not mean they are physically reasonable. For example, when we apply the theory of Israel and Stewart to thermal diffusion, we obtain so called telegraph equation. A telegraph equation is reduced to wave equation in high speed (highly relativistic) limit; it does not violate causality but wave equation does not represent diffusion.

A method proposed here is to solve the evolution of the particle distribution function, which is defined on the spacetime (x,t), along the proper time. The evolution cannot be formulated in the form of diffusion equation along propertime because the direction of time is forward only. To avoid this problem the method of path integrals with the constraint of energy shell is introduced in the present study.

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